Concepts: Interval Notation, Linear Inequalities, Compound Inequalities, Absolute Value Inequalities.

The solution to an inequality can be an infinite set of points, and so we need to understand how to represent a set of points mathematically. We will often use a number line to draw a visual representation.

When sketching an inequality you use an open circle if the endpoint is not included, and a filled in circle if the endpoint is included. Here's how you can remember this:

For $<$ (one thing)	draw $\circ$	one thing (draw the circle)
For $>$ (one thing)	draw $\circ$	one thing (draw the circle)
For $\geq$ (two things)	draw $\bullet$	two things (draw the circle and then shade it in)
For $\leq$ (two things)	draw $\bullet$	two things (draw the circle and then shade it in)

Set Notation and Interval Notation for Inequalities

$\{x \vert a \leq x \leq b\}$	is equivalent to	$x \,\epsilon\, [a, b]$ (closed interval)
$\{x \vert a < x < b\}$		$x \,\epsilon\, (a, b)$ (open interval)
$\{x \vert a \leq x < b\}$		$x \,\epsilon\, [a, b)$ (half open or half closed interval)
$\{x \vert a < x \leq b\}$		$x \,\epsilon\, (a, b]$ (half open or half closed interval)
$\{x \vert x \geq a\}$		$x \,\epsilon\, [a, \infty)$ (closed interval)
$\{x \vert x \leq a\}$		$x \,\epsilon\, (-\infty, a]$ (closed interval)
$\{x \vert x > a\}$		$x \,\epsilon\, (a, \infty)$ (open interval)
$\{x \vert x < a\}$		$x \,\epsilon\, (-\infty, a)$ (open interval)
$\{x \vert x \in \mathbb{R}\}$		$x \,\epsilon\, (-\infty, \infty)$ (open interval)

The algebraic process of solution of a linear *inequality* is the same as for an equation, except that
the inequality is reversed if you multiply or divide by a negative number.

Example Solve $\frac{3x+5}{4} + \frac{7}{12} > -\frac{x}{6}$.

Let's start this one by clearing fractions. So we use the Multiplication Principle with the factor 12 (which is the LCD). Since 12 is positive, we don't change the direction of the inequality.

$$\begin{aligned}
\frac{3x+5}{4} + \frac{7}{12} &> -\frac{x}{6} \\
12 \times \left(\frac{3x+5}{4} + \frac{7}{12}\right) &> 12 \times \left(-\frac{x}{6}\right) \text{ (now distribute the factor of 12)} \\
3(3x+5) + 7 &> -2x \text{ (distribute the 3)} \\
9x + 15 + 7 &> -2x \text{ (simplify)} \\
9x + 22 &> -2x \\
9x + 22 - 9x &> -2x - 9x \text{ (Use Additive Principle)} \\
22 &> -11x \\
&\quad \text{(Use Multiplication Principle to isolate the } x, \\
&\quad \text{ since we are multiplying by a negative number change direction of inequality)} \\
\frac{1}{-11} 22 &< \frac{1}{-11}(-11x) \text{ (simplify)} \\
-2 &< x \text{ (simplify)}
\end{aligned}$$

Example Solve $5(x-3) \leq 2(x-3)$.

The goal is still to first isolate a single term with the x in it on one side of the equation. If we multiply or divide by a negative number, we must switch the direction of the inequality.

$$\begin{aligned}
5(x-3) &\leq 2(x-3) \\
5x - 15 &\leq 2x - 6 \\
5x - \cancel{15} + \cancel{15} &\leq 2x - 6 + 15 \\
5x &\leq 2x + 9 \\
5x - 2x &\leq \cancel{2x} + 9 - \cancel{2x} \\
3x &\leq 9 \\
\frac{1}{3} \times 3x &\leq \frac{1}{3} \times 9 \\
x &\leq 3
\end{aligned}$$

Solving Absolute Value Equalities and Inequalities: Three Cases

Case1 For equalities of the form $\left|ax+b\right| = \left|cx+d\right|$, the solution is

$$ax + b = cx + d \qquad \text{or} \qquad ax + b = -(cx + d).$$

The solution will be two distinct numbers.

Case2 For inequalities of the form $\left|ax+b\right| < c$, where $c > 0$ the solution is

$$-c < ax + b < c.$$

NOTE: it is important that the $-c$ is on the left and the c is on the right. If this isn't the case, you will get the wrong solution.

The solution will be a set of points between two numbers.

Case3 For inequalities of the form $\left|ax+b\right| > c$, where $c > 0$ the solution is

$$ax + b < -c \qquad \text{or} \qquad ax + b > c.$$

The solution will be a set of numbers less than one number or greater than another number.

Example Reduce $|x| < 6$.

Interval notation: $-6 < x < 6$
Set notation: $x \in (-6, 6)$

Example Reduce $|2x - 5| \leq 7$.

$$-7 \leq 2x - 5 \leq 7$$
$$-2 \leq 2x \leq 12$$
$$-1 \leq x \leq 6$$

Interval notation: $-1 \leq x \leq 6$
Set notation: $x \in [-1, 6]$