Concepts: Functions, Functional Notation, Graphs, Domain & Range.

Functions and Their Properties

These are concepts you will get practice with throughout this course, as we study and learn properties for specific functions. When you get to calculus, the concepts continuity, increasing/decreasing, extrema, asymptotes, end behaviour will be discussed using the ideas of calculus (limits and derivatives).

A *function* f is a rule that assigns to each element x in a set D exactly one element, called $f(x)$, in a set R.

The *range* R is the set of all possible values of $f(x)$, when x varies over the entire *domain* D.

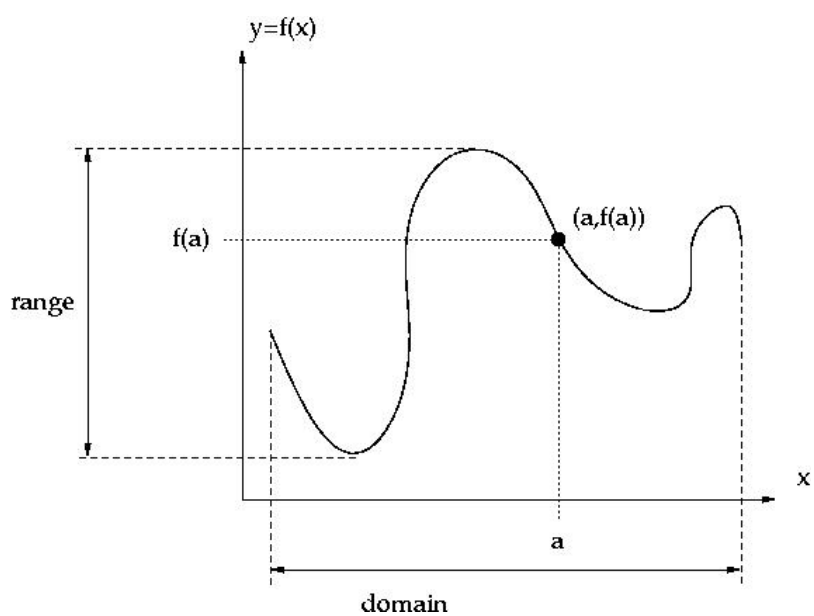
The functions we consider have the domain and range as subsets of the real numbers. The real numbers are denoted $(-\infty, \infty)$ or $\mathbb{R}$.

We often use $y = f(x)$ as dependent variable (it's called dependent because it depends on the value of x). This notation is called *Euler's function notation*. This is read as "y equals f of x". Note that this is not multiplication, that is, $f(x)$ does <u>not</u> mean f times x.

Graph of $y = f(x)$: A graph of $y = f(x)$ pictorially represents the relationship between ordered pairs, where the first element in the pair is the domain, the second element the range:

$$\{(x, f(x)) | x \in D\}$$

read: "the set of ordered pairs $(x, f(x))$ such that x is an element of D which is the domain." The graph contains more information than the other descriptions. One of the goals in this class is to be able to sketch the graphs of functions by hand.



More on Domain and Range Given $y = f(x)$, the values of x that can go into $f(x)$ and yield an output which is a real number form the domain. All the possible y's that come out form the range.

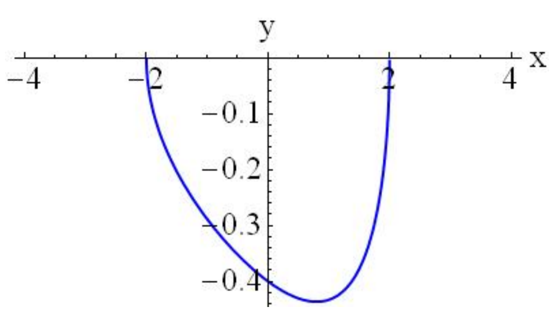
Example Find the domain and range of $h(x) = \dfrac{\sqrt{4-x^2}}{x-5}$.
We cannot have division by zero, so we want to see where the denominator is zero and exclude that value of x from the domain:

$$\begin{aligned} x - 5 &= 0 \\ x &= 5 \end{aligned}$$

so $x = 5$ is not in the domain.
We also cannot take the square root of a negative number and get a result which is a real number. So we must have values of x for which

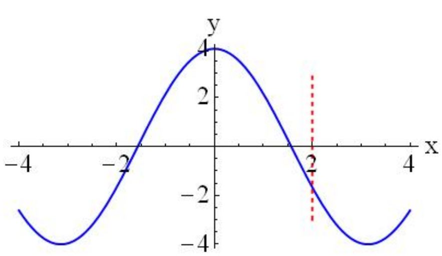
$$\begin{aligned} 4 - x^2 &\geq 0 \text{ (We'll see a cool way of simplifying inequalities later–for now, use our number sense.)} \\ 4 &\geq x^2 \\ x^2 &\leq 4 \\ -2 \leq x &\leq 2 \end{aligned}$$

This means the domain of the function $h(x)$ is $-2 \leq x \leq 2$, or $x \in [-2, 2]$. The point $x = 5$ is excluded, but that is already contained in the restriction based on the square root.
The range is all possible output values. This is usually more complicated to figure out than the domain, but easy to find if we plot a graph using a computer of calculator:

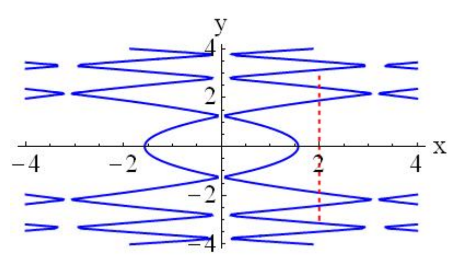


From the graph, we estimate the range to be $y \in [-0.44, 0]$. To get the range precisely we could use ideas from calculus, or other ideas we will develop later.

Vertical Line Test A graph represents a function if every vertical line you can draw intersects the graph only once (this ensures we have exactly one element $f(x)$ for each x).

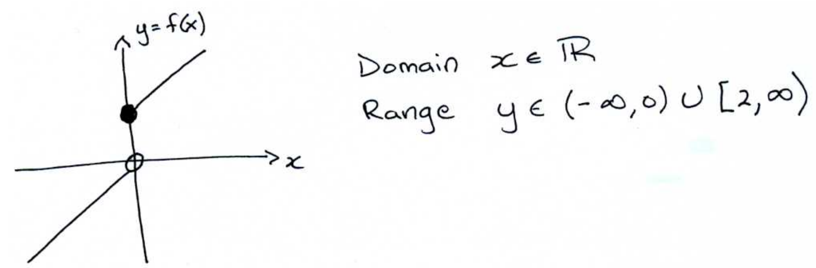


this graph represents a function



this graph does not represent a function

Example Find the domain and range of $f(x) = \begin{cases} x \text{ if } x < 0 \\ x + 2 \text{ if } x \geq 0 \end{cases}$



Read the section in the text on continuity; this concept is developed in greater detail in calculus. The precise definition of continuous requires a precise definition of the *limit of a function*,

$$\lim_{x \to a} f(x).$$

For now, think of this limit as the words "the value of $f(x)$ as x approaches a". The concept of limit is where a calculus course begins. We will be talking more about limits this semester, to help you get used to the notation.