Concepts: Piecewise Function, Continuity, Increasing & Decreasing, Bounded, Extrema.

Some Basic Functions

One of the goals when studying functions is to be able to identity basic functions quickly. Here are four basic functions that you should be very familiar with:

- The Square Function $f(x) = x^2$
- The Square Root Function $f(x) = \sqrt{x}$
- The Cube Function $f(x) = x^3$
- The Cube Root Function $f(x) = \sqrt[3]{x}$

The text has an excellent discussion of all these as well as the semicircles:

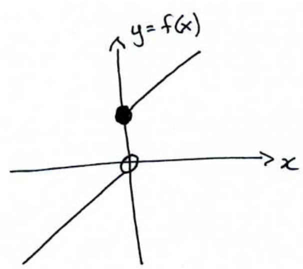
- Top Semicircle Function $f(x) = \sqrt{r^2 - x^2}$
- Bottom Semicircle Function $f(x) = -\sqrt{r^2 - x^2}$

Be sure that you can quickly and correctly sketch all these, as well as determine the domain and range.

Piecewise Function

A *piecewise function* is a function that has different formulas over different parts of the domain.

Example Find the domain and range of $f(x) = \begin{cases} x \text{ if } x < 0 \\ x + 2 \text{ if } x \geq 0 \end{cases}$



Domain $x \in \mathbb{R}$

Range $y \in (-\infty, 0) \cup [2, \infty)$

Notice we use the open/closed circle notation we saw previously when looking at inequalities.

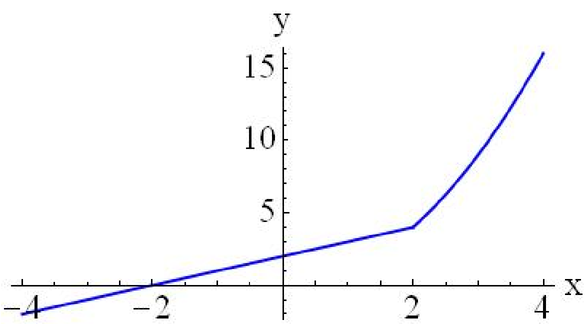
If there is a closed circle corresponding to the value $x = a$, then the point $(a, f(a))$ is part of the function definition. An open circle at the point $(a, f(a))$ means that point is not part of the function definition.

In the graph above, the value of the function at $x = 0$ is $y = 2$, not $y = 0$.

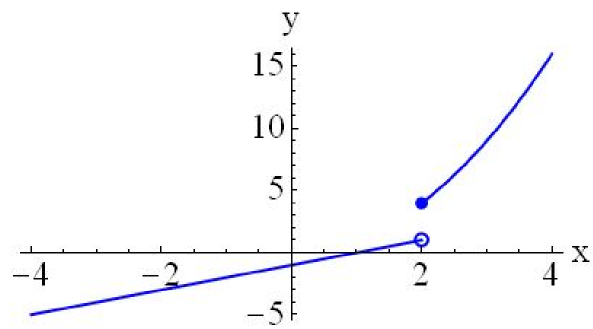
Continuity

Continuity is important in calculus, but it is not discussed in our text since to describe continuity correctly requires the concept of limits which are introduced in calculus. I believe it is useful to have a simple understanding of continuity before you see it in calculus, so I will introduce it here.

Graphically, a *continuous* function can be drawn without lifting your pen.



this graph represents a continuous function



this graph does not represent a continuous function

The precise definition of continuous requires a precise definition of the *limit of a function*,

$$\lim_{x \to a} f(x).$$

For now, think of this limit as the words "the value of $f(x)$ as x approaches a". The concept of limit is where a calculus course begins. We will be talking more about limits this semester, to help you get used to the notation.

Boundedness

Another concept not discusses in the text which I may mention from time to time is *bounded*, so I include it here.

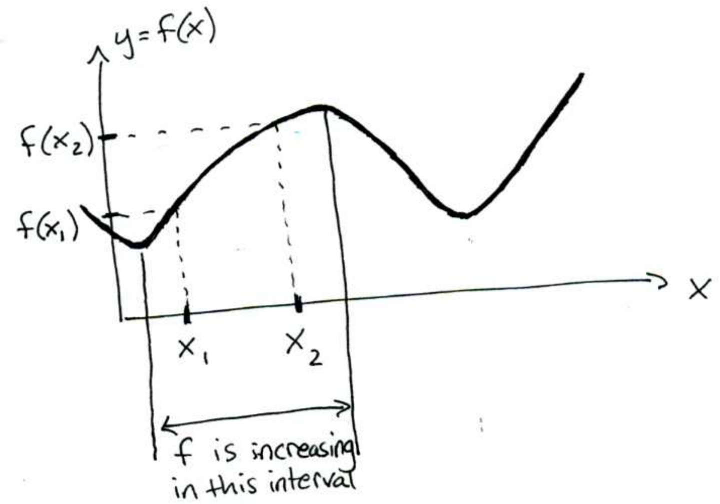
A function f is

- bounded below if there is a number b that is less than or equal to every other number in the range of f.
- bounded above if there is a number B that is greater than or equal to every other number in the range of f.
- bounded if it is bounded above and below.

Increasing and Decreasing Functions

A function is *increasing* on an Interval I if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.

A function is *decreasing* on an Interval I if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in I.



The first derivative of a function, which you will learn about in calculus, gives you information on whether a function is increasing or decreasing without having to look at a sketch.

Local and Absolute Extrema

A function can have peaks and valleys; the value of the function at the peak is called a maximum, the value of the function at a valley is called a minimum. These extreme values of the function are called <u>extrema</u>.

Local extrema are peaks and valleys only in a local area; absolute extrema are the maximum and minimum value of the function over the entire range of f.

These are easy to spot from a graph. In calculus, you will learn a way to determine extrema without looking at a graph of a function.