You should be *expanding* this study guide as you see fit with details and worked examples. With this extra layer of detail you will then have excellent study notes for exams, and later reference.

Some topics will be emphasized more than others.

Practice is suggested from Dugopolski *Precalculus, functions and graphs*, 4th Edition.

B.1 Real Numbers and Their Properties (week of aug 24)

- natural numbers $\{1, 2, 3, 4, \ldots\}$
- whole numbers $\{0, 1, 2, 3, \ldots\}$
- integer numbers $\{\ldots -3, -2, -1, 0, 1, 2, 3, \ldots\}$
- rational numbers: numbers of form $\frac{a}{b}$ where a and b are integers and $b \neq 0$
- irrational numbers: numbers like π, e, $\sqrt{2}$
- real numbers $\mathbb{R}$: all the above numbers together make up the real numbers
- properties of absolute value
- order of operations
- Practice: B.1.25-30, B.1.38, B.1.69

B.2 Exponents and Radicals (week of aug 24)

- rules of exponents

  $a^0 = 1$ if $a \neq 0$
  $a^m a^n = a^{m+n}$
  $\frac{a^m}{a^n} = a^{m-n}$
  $(a^m)^n = a^{mn}$
  $(ab)^n = a^n b^n$
  $(\frac{a}{b})^n = \frac{a^n}{b^n}$
  $a^{1/n} = \sqrt[n]{a}$
  $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$ if $a, b \in \mathbb{R}$ and n is positive integer

- rationalizing denominator: multiply numerator and denominator by conjugate of denominator
- rationalizing numerator: multiply numerator and denominator by conjugate of numerator
- Practice: B.2.24, B.2.44, B.2.67, B.2.91

B.3 Polynomials (week of aug 24)

- polynomial $a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$, n is a whole number, $a_0, a_1, \ldots a_n \in \mathbb{R}$
- degree of a polynomial (largest power of x)
- leading coefficient a_n
- addition and subtraction of polynomials by collecting like terms
- multiplying polynomials using distributive property
- special products

  $(a+b)^2 = a^2 + 2ab + b^2$
  $(a-b)^2 = a^2 - 2ab + b^2$
  $(a+b)(a-b) = a^2 - b^2$

- long division of polynomials
- Practice: B.3.16, B.3.29, B.3.31, B.3.47, B.3.55, B.3.67, B.3.71

B.4 Factoring Polynomials (WEEK OF AUG 24)

- factoring by grouping
- factoring special products

  $a^2 + 2ab + b^2 = (a + b)^2$
  $a^2 - 2ab + b^2 = (a - b)^2$
  $a^2 - b^2 = (a + b)(a - b)$
  $a^3 - b^3 = (a - b)(a^2 + 1b + b^2)$
  $a^3 + b^3 = (a + b)(a^2 - 1b + b^2)$

- Practice: B.4.7, B.4.15, B.4.31, B.4.45, B.4.55

B.5 Rational Expressions (avoid cases where denominator would be zero) (WEEK OF AUG 31)

- $\dfrac{ac}{bc} = \dfrac{a}{b}$ (This is a very important property-you can only cancel *factors*!)
- $\dfrac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \dfrac{a}{b} \cdot \dfrac{d}{c}$ (invert and multiply)
- addition and subtraction requires a common denominator; multiplication and division do not
- Practice: B.5.4, B.5.25, B.5.31, B.5.47, B.5.50, B.5.55, B.5.58, B.5.59

1.1 Equations in One Variable (WEEK OF AUG 31)

- equivalent expressions, and equivalent equations.
- properties of equality ($A + B = C + D$ means $W(A + B) = W(C + D)$ if $W \neq 0$)

  conditional equation (true for some x)
  inconsistent equation (not true for any x)
  identity (true for all x)

- equation with rational expressions: multiply by Lowest Common Denominator, check what you found is a solution
- absolute values

  $|x| = k$ ($k > 0$), is equivalent to $x = -k$ or $x = k$ (compound statement)
  $|x| = 0$ is equivalent to $x = 0$
  $|x| = k$ ($k < 0$) has no solution

- Practice: 1.127-32, 1.1.39, 1.1.59, 1.1.69, 1.1.71, 1.1.91

1.2 Constructing Models to Solve Problems (WEEK OF AUG 31)

- some word problems, and constructing models from English phrases
- Practice: 1.2.9, 1.2.13, 1.2.17, 1.2.43, 1.2.44, 1.2.52, 1.2.55, 1.2.56, 1.2.60

1.3 Equations and Graphs in Two Variables (WEEK OF AUG 31)

- the Cartesian coordinate system (xy-plane), ordered pairs (x_1, y_1) and (x_2, y_2)
- midpoint formula $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$
- distance formula $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
- circle $(x - h)^2 + (y - k)^2 = r^2$, center (h, k), radius r
- completing the square
- standard form of equation of straight line $Ax + By + C = 0$, where $A, B, C \in \mathbb{R}$.
- Practice: 1.3.21, 1.3.31, 1.3.36, 1.3.47, 1.3.57, 1.3.64, 1.3.71, 1.3.89

1.4 Linear Equations in Two Variables (WEEK OF AUG 31)

- slope $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$
- point-slope form $y - y_1 = m(x - x_1)$
- slope-intercept form $y = mx + b$
- parallel lines have same slope
- perpendicular lines have slopes that are negative reciprocal
- Practice: 1.4.19, 1.4.33, 1.4.57, 1.4.67, 1.4.71, 1.4.84

1.6 Complex Numbers (WEEK OF SEP 7)

- $i = \sqrt{-1}$ or $i^2 = -1$
- complex numbers have form $a + bi$ where $a, b \in \mathbb{R}$
- add/subtract complex numbers by collecting real part and imaginary part separately
- multiply complex numbers by using distributive property and $i^2 = -1$
- divide complex numbers by multiplying numerator and denominator by complex conjugate of denominator
- $\sqrt{-b} = i\sqrt{b}$ for $b \in \mathbb{R}$
- Practice: 1.6.29, 1.6.41, 1.6.53, 1.6.61, 1.6.91

1.7 Quadratic Equations (WEEK OF SEP 7)

- zero factor property: $AB = 0$ is equivalent to $A = 0$ or $B = 0$ (compound statement)
- quadratic equation is $ax^2 + bx + c = 0$
- solve by

  factoring (works if you can factor using grouping method)
  the square root property (works if $b = 0$)
  complete the square (always works)
  the quadratic formula (always works)

- memorize quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- discriminant is $b^2 - 4ac$ tells you what kinds of solutions to expect
- Pythagorean Theorem $a^2 + b^2 = c^2$ for right triangle with length of sides a, b and hypotenuse c
- 1.7.23, 1.7.29, 1.7.40, 1.7.58, 1.7.71, 1.7.83

1.8 Linear and Absolute Value Inequalities (WEEK OF SEP 7)

- interval notation (know how to convert between these)

  set notation $\{x|x > a\}$
  interval notation $x \in (-\infty, 14)$ (smaller value on left) $(,)$ does not include endpoint; $[,]$ does
  number line notation

- linear inequalities
- compound inequalities (simplify by drawing number lines)
- intersection $x \in (-3, 5] \cap [4, \infty) = [4, 5]$ (intersections can be simplified) (x must satisfy both)
- union $x \in (-3, 5] \cup (12, 17)$ (x may satisfy either)
- absolute value inequalities ($k > 0$)

  $|x| > k$ is equivalent to $x < -k$ or $x > k$
  $|x| \geq k$ is equivalent to $x \leq -k$ or $x \geq k$
  $|x| < k$ is equivalent to $-k < x < k$
  $|x| \leq k$ is equivalent to $-k \leq x \leq k$

- Practice: 1.8.18, 1.8.21, 1.8.25, 1.8.49, 1.8.51, 1.8.53, 1.8.59, 1.8.65, 1.8.85, 1.8.96

2.1 Functions (WEEK OF SEP 14)

- relations and functions (vertical line test)
- domain (x values) and range (y values)
- functional notation $y = f(x)$ (x continuous); sets of ordered pairs such as $\{(2, 1), (4, 12)\}$ (x discrete)
- average rate of change (difference quotient) $\frac{\Delta y}{\Delta x} = \frac{f(x+h)-f(x)}{h}$
- Practice: 2.1.2, 2.1.34, 2.1.17-22, 2.1.65-76, 2.1.90, 2.1.94, 2.1.100

2.2 Graphs of Relations and Inverses (WEEK OF SEP 21)

- the graph of a relation
- the squaring function $f(x) = x^2$
- the square root function $f(x) = \sqrt{x}$
- the cubing function $f(x) = x^3$
- the cube root function $f(x) = \sqrt[3]{x} = x^{1/3}$
- semicircles:

  $f(x) = \sqrt{r^2 - x^2}$ (top)
  $f(x) = -\sqrt{r^2 - x^2}$ (bottom)

- piecewise functions (defining, graphing)

  the greatest integer function $f(x) = \lfloor x \rfloor$
  the absolute value function $f(x) = |x|$

- increasing, decreasing, or constant functions (on an interval)
- Practice: 2.2.9, 2.2.42, 2.2.46, 2.2.49-56, 2.2.65

2.3 Families of Functions, Transformations, and Symmetry (WEEK OF SEP 21)

- transformations (graphical and algebraic) of $y = f(x)$ where $h > 0$, $k < 0$, $c > 1$:

  horizontal shift to right: $y = f(x - h)$, $h > 0$
  horizontal shift to left: $y = f(x + h)$, $h > 0$
  vertical shift up: $y = f(x) + k$, $k > 0$
  vertical shift down: $y = f(x) - k$, $k > 0$
  reflect about x axis: $y = -f(x)$
  reflect about y axis: $y = f(-x)$
  stretch vertically: $y = cf(x)$, $c > 1$
  compress horizontally: $y = f(cx)$, $c > 1$
- notice: anything outside of f is vertical, anything inside of f is horizontal and opposite
- multiple translations, for example $y = a(x - h)^2 + k$
- Practice: 2.3.9, 2.3.27-34, 2.3.60, 2.3.62, 2.3.73, 2.3.104, 2.3.106

2.4 Operations with Functions (WEEK OF SEP 28)

- sum $(f + g)(x) = f(x) + g(x)$
- difference $(f - g)(x) = f(x) - g(x)$
- product $(fg)(x) = f(x)g(x)$
- quotient $(f/g)(x) = f(x)/g(x)$
- composition $(f \circ g)(x) = f(g(x))$
- determine domain by looking at unsimplified form (division by zero, square root of negative)
- domain often requires solving compound inequalities
- note $f \circ g \neq g \circ f$ (not commutative)
- Practice: 2.4.23-30, 2.4.50, 2.4.70, 2.4.81, 2.4.78

2.5 Inverse Functions (WEEK OF SEP 28)

- definition of one-to-one functions (horizontal line test)
- inverse functions (both graphically, and algebraically)
- notation for inverse function: $f^{-1}(x) \neq \frac{1}{f(x)}$
- check using $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$
- Practice: 2.5.36, 2.5.50, 2.5.54, 2.5.58, 2.5.61, 2.5.74, 2.5.96

2.6 Constructing Functions with Variation (WEEK OF OCT 5)

- direct variation $y = kx$
- inverse variation $y = \frac{k}{x}$
- joint variation $y = kxz$
- combined variation
- sketch of $y = x^m$ where m is an integer
- Practice: 2.6.10, 2.6.38, 2.6.54

3.1 Quadratic Functions and Inequalities (week of oct 12)

- vertex form $f(x) = a(x-h)^2 + k$ has vertex (h, k), opens up if $a > 0$ down if $a < 0$
- sketching quadratics
- sign charts to solve quadratic inequalities
- Practice: 3.1.6, 3.1.17, 3.1.25, 3.1.35, 3.1.53, 3.1.66, 3.1.80

3.2 Zeros of Polynomial Functions (week of oct 12)

- Remainder Theorem: the remainder when $P(x)$ is divided by $x - c$ is $P(c)$
- Factor Theorem: c is a zero of $y = P(x)$ is and only if $x - c$ is a factor of $P(x)$
- Rational Zero Theorem: if p/q is a rational zero in lowest terms for $y = P(x)$ (integer coefficients), p is a factor of constant term, q is a factor of leading coefficient
- Fundamental Theorem of Algebra: $P(x)$ is a polynomial of positive degree, then $y = P(x)$ has at least one zero in the set of complex numbers
- Practice: 3.2.2, 3.2.5, 3.2.35, 3.2.42, 3.2.47, 3.2.61, 3.2.73

3.3 The Theory of Equations (week of oct 12)

- multiplicity: $P(x) = (x+1)^4$ has a root of $x = -1$ with multiplicity 4
- Descartes's Rule of Signs: changes of signs of $P(x)$ and $P(-x)$ tells you something about the roots of $P(x)$
- Practice: 3.3.3, 3.2.43, 3.2.61

3.4 Miscellaneous Equations (week of oct 19)

- factoring higher degree equations
- equations involving square roots
- equations with rational exponents
- equations of quadratic type
- equations involving absolute values
- Practice: 3.4.9, 3.4.2, 3.4.25, 3.4.30, 3.4.35, 3.4.42, 3.4.61, 3.4.73, 3.4.91

3.5 Graphs of Polynomial Functions (week of oct 26)

- Intermediate Value Theorem
- sketching polynomials

  x-intercept with multiplicity
  end behavior $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$
- solving polynomial inequalities using sign charts
- Practice: 3.5.1, 3.5.18, 3.5.53, 3.5.63, 3.5.83, 3.5.91, 3.5.97

3.6 Rational Functions and Inequalities (WEEK OF OCT 26)

- horizontal asymptote $y = L$: $\lim_{x\to\infty} f(x) = L$ or $\lim_{x\to-\infty} f(x) = L$
- vertical asymptotes $x = a$: $\lim_{x\to a^+} f(x) = \pm\infty$ or $\lim_{x\to a^-} f(x) = \pm\infty$
- slant asymptotes $y = mx + b$: $\lim_{x\to\infty} f(x) = mx + b$ or $\lim_{x\to-\infty} f(x) = mx + b$
- sketching rational functions

  x-intercepts and multiplicity
  vertical asymptotes and multiplicity
  end behavior $\displaystyle\lim_{x\to\infty} f(x)$ and $\displaystyle\lim_{x\to-\infty} f(x)$
- solving rational inequalities using sign charts
- Practice: 3.6.2, 3.6.14, 3.6.30, 3.6.52, 3.6.73-80, 3.6.113, 3.6.132

4.1 Exponential Functions and Their Applications (WEEK OF NOV 9)

- definition of $f(x) = a^x$ and $f(x) = e^x$, $f(x) = e^{-x}$
- domain and range
- graphing and graphical transformations
- simple exponential equations
- compound interest and radioactive decay
- Practice: 4.1.43, 4.1.45, 4.1.61, 4.1.97, 4.1.119

4.2 Logarithmic Functions and Their Application (WEEK OF NOV 9)

- definition of $f(x) = \log_a(x)$ and $f(x) = \ln(x)$
- domain and range
- graphing and graphical transformations
- simple logarithmic and exponential equations
- inverse functions
- Practice: 4.2.21, 4.2.43, 4.2.55, 4.2.57, 4.2.103, 4.2.133

4.3 Rules of Logarithms (WEEK OF NOV 9)

- Memorize the rules of exponents and logarithms:

  $a^0 = 1$ if $a \neq 0$
  $a^m a^n = a^{m+n}$ (product same base)
  $\frac{a^m}{a^n} = a^{m-n}$ (quotient same base)
  $(a^m)^n = a^{mn}$ (power)
  $(ab)^n = a^n b^n$ (product different base)
  $(\frac{a}{b})^n = \frac{a^n}{b^n}$ (quotient different base)
  $a^{1/n} = \sqrt[n]{a}$ (fractional exponent)
  $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$ if $a, b \in \mathbb{R}$ and n is positive integer (radical notation)
  $\ln e^x = x$ and $e^{\ln x} = x$ (inverse)
  $\ln(MN) = \ln(M) + \ln(N)$ (product)
  $\ln(\frac{M}{N}) = \ln(M) - \ln(N)$ (quotient)
  $\ln(M^N) = N\ln(M)$ (power)
- change of base in exponential
- Practice: 4.3.13, 4.3.18, 4.3.25, 4.3.51, 4.3.84, 4.3.101

4.4 More Equations and Applications (WEEK OF NOV 16)

- solving exponential equations
- solving logarithmic equations
- applications: radioactive dating, Newton's law of cooling, loans
- Practice: 4.4.5, 4.425, 4.4.27, 4.4.43, 4.4.46, 4.4.51, 4.4.56, 4.4.71, 4.4.73, 4.4.86

8.1 Systems of Linear Equations in Two Variables (WEEK OF NOV 30)

- graphical solution
- algebraic solution

  substitution method
  elimination method (addition method)

- three possibilities

  independent (exactly one solution)
  dependent (infinite number of solutions)
  inconsistent (no solutions)

- Practice: 8.1.9, 8.1.11, 8.1.25, 8.1.27, 8.1.37, 8.1.42

8.2 Systems of Linear Equations in Three Variables (WEEK OF NOV 30)

- no graphical solution (typically hard to sketch in $\mathbb{R}^3$)
- algebraic solution

  substitution method
  elimination method (addition method)

- three possibilities

  independent (exactly one solution)
  dependent (infinite number of solutions)
  inconsistent (no solutions)

- Practice: 8.2.11, 8.2.15, 8.2.27

8.3 Nonlinear Systems of Equations (WEEK OF NOV 30)

- graphical solution
- algebraic solution

  substitution method

- solve by elimination of variables
- Practice: 8.3.21, 8.3.23, 8.3.61

8.5 Inequalities and Systems of Inequalities in Two Variables (WEEK OF NOV 30)

- graphical solution
- no algebraic solution (Calculus III: if you need it, from sketch write the description of the region)
- Practice: 8.5.1-4, 8.5.22, 8.5.37, 8.5.51

10.1 The Parabola (WEEK OF NOV 30)

- our focus is sketching
- definition and general form
- opening up/down: $y = a(x-h)^2 + k$ vertex (h, k), focus $(h, k+p)$, directrix $y = k - p$
- opening up/down: $x = a(y-k)^2 + h$ vertex (h, k), focus $(h+p, k)$, directrix $x = h - p$
- sketching
- completing the square
- Practice: 10.1.5-10, 10.1.47, 10.1.71

10.2 The Ellipse and the Circle (WEEK OF DEC 7)

- our focus is sketching
- definition and general form
- circle $(x-h)^2 + (y-k)^2 = r^2$ center (h, k), radius r
- ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ center (h, k)
- sketching using a box (inside box)
- completing the square
- Practice: 10.2.5-8, 10.3.38, 10.2.63

10.3 The Hyperbola (WEEK OF DEC 7)

- our focus is sketching
- definition and general form
- hyperbola opens left/right: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$, $b^2 = c^2 - a^2$, foci $(\pm c, 0)$
- hyperbola opens up/down: $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$, $b^2 = c^2 - a^2$, foci $(0, \pm c)$
- note a is under positive term in both cases
- sketching using a box (outside box)
- completing the square
- Practice: 10.3.5-8, 10.3.17, 10.3.24