

Questions

1. Solve for x when $\frac{2}{3}x = \frac{1}{15}x + \frac{3}{5}$.
2. Solve for x when $\frac{x}{2} + \frac{x}{5} = \frac{7}{10}$.
3. Solve for x when $20 - \frac{1}{3}x = \frac{1}{2}x$.
4. Is 4 a solution to $\frac{1}{2}(y - 2) + 2 = \frac{3}{8}(3y - 4)$?
5. Solve for x when $\frac{4}{5}x - \frac{2}{3} = \frac{3x + 1}{2}$.
6. Solve for x when $-1 + 5(x - 2) = 12x + 3 - 7x$.
7. Solve $\frac{8}{x} + \frac{2}{5} = -\frac{2}{x}$.
8. Solve $\frac{x + 1}{2x} = \frac{2}{3}$.
9. Solve $\frac{2}{2x + 5} = \frac{4}{x - 4}$.
10. Solve $\frac{3}{x + 5} = \frac{3}{3x - 2}$.
11. Solve $7 - \frac{x}{x + 5} = \frac{5}{x + 5}$.
12. Solve $\frac{8x}{4x^2 - 1} = \frac{3}{2x + 1} + \frac{3}{2x - 1}$.
13. Solve $\frac{6}{x - 3} = \frac{-5}{x - 2} - \frac{5}{x^2 - 5x + 6}$.
14. Reduce $|x - 6| = 16$.
15. Reduce $|2x - 5| = 13$.
16. Reduce $|\frac{1}{2} - \frac{3}{4}x| + 1 = 3$.
17. Reduce $|4 - \frac{5}{2}x| = 12$.
18. Reduce $|x + 6| = |2x - 3|$.

Solutions

1. The LCD (lowest common denominator) is 15, so multiply the equation by 15 to remove the fractions.

$$\begin{aligned} \frac{2}{3}x &= \frac{1}{15}x + \frac{3}{5} \\ 15 \cdot \left(\frac{2}{3}x\right) &= 15 \cdot \left(\frac{1}{15}x + \frac{3}{5}\right) \\ 10x &= 15 \cdot \frac{1}{15}x + 15 \cdot \frac{3}{5} \text{ distribute!} \\ 10x &= x + 9 \text{ simplify} \\ 10x - x &= x + 9 - x \text{ addition principle} \\ 9x &= 9 \text{ simplify} \\ \frac{1}{9} \cdot 9x &= \frac{1}{9} \cdot 9 \text{ multiplication principle} \\ x &= 1 \text{ simplify} \end{aligned}$$

2. LCD is 10.

$$\begin{aligned} \frac{x}{2} + \frac{x}{5} &= \frac{7}{10} \\ 10 \cdot \left(\frac{x}{2} + \frac{x}{5}\right) &= 10 \cdot \frac{7}{10} \\ 10 \cdot \frac{x}{2} + 10 \cdot \frac{x}{5} &= 7 \\ 5x + 2x &= 7 \\ 7x &= 7 \\ \frac{1}{7} \cdot 7x &= \frac{1}{7} \cdot 7 \\ x &= 1 \end{aligned}$$

3. LCD is 6.

$$\begin{aligned} 20 - \frac{1}{3}x &= \frac{1}{2}x \\ 6 \cdot \left(20 - \frac{1}{3}x\right) &= 6 \cdot \frac{1}{2}x \\ 6 \cdot 20 - 6 \cdot \frac{1}{3}x &= 3x \\ 120 - 2x &= 3x \\ 120 - 2x + 2x &= 3x + 2x \\ 120 &= 5x \\ \frac{1}{5} \cdot 120 &= \frac{1}{5} \cdot 5x \\ 24 &= x \end{aligned}$$

4. You could substitute $y = 4$ to check, but I am going to solve it instead. LCD is 8.

$$\begin{aligned}\frac{1}{2}(y-2) + 2 &= \frac{3}{8}(3y-4) \\ 8 \cdot \left(\frac{1}{2}(y-2) + 2\right) &= 8 \cdot \frac{3}{8}(3y-4) \\ 8 \cdot \frac{1}{2}(y-2) + 8 \cdot 2 &= 3(3y-4) \\ 4(y-2) + 16 &= 9y-12 \\ 4y-8+16 &= 9y-12 \\ 4y+8 &= 9y-12 \\ 4y+8-9y-8 &= 9y-12-9y-8 \\ -5y &= -20 \\ \frac{1}{-5} \cdot (-5y) &= \frac{1}{-5} \cdot (-20) \\ y &= 4\end{aligned}$$

5. LCD is 30.

$$\begin{aligned}\frac{4}{5}x - \frac{2}{3} &= \frac{3x+1}{2} \\ 30 \cdot \left(\frac{4}{5}x - \frac{2}{3}\right) &= 30 \cdot \frac{3x+1}{2} \\ 30 \cdot \frac{4}{5}x - 30 \cdot \frac{2}{3} &= 30 \cdot \frac{1}{2} \cdot (3x+1)\end{aligned}$$

Note in above I wrote $\frac{3x+1}{2}$ as $\frac{1}{2} \cdot (3x+1)$. Doing this helps reduce errors!

$$\begin{aligned}24x-20 &= 15 \cdot (3x+1) \\ 24x-20 &= 45x+15 \\ 24x-20-45x+20 &= 45x+15-45x+20 \\ -21x &= 35 \\ \frac{1}{-21} \cdot (-21x) &= \frac{1}{-21} \cdot 35 \\ x &= -\frac{35}{21} = -\frac{5}{3}\end{aligned}$$

6.

$$\begin{aligned}-1+5(x-2) &= 12x+3-7x \\ -1+5x-10 &= 5x+3 \\ 5x-9-5x &= 5x+3-5x \\ -9 &= 3\end{aligned}$$

We have to interpret what we have found. Since -9 never equals 3 , the equation is never true no matter what value of x we put in. This means the equation has no solution. It is an inconsistent equation.

7. Lowest common denominator is $5x$.

$$\begin{aligned} \left(\frac{8}{x}\right)5x + \left(\frac{2}{5}\right)5x &= \left(-\frac{2}{x}\right)5x \\ 40 + 2x &= -10 \\ 2x &= -10 - 40 \\ x &= -\frac{50}{2} = -25 \end{aligned}$$

Check:

$$\begin{aligned} \frac{8}{(-25)} + \frac{2}{5} &= -\frac{2}{(-25)} \\ -\frac{8}{25} + \frac{10}{25} &= \frac{2}{25} \\ \frac{2}{25} &= \frac{2}{25} \text{ it's a solution} \end{aligned}$$

8. LCD is $6x$.

$$\begin{aligned} \left(\frac{x+1}{2x}\right)6x &= \left(\frac{2}{3}\right)6x \\ 3x + 3 &= 4x \\ 3 &= 4x - 3x \\ 3 &= x \end{aligned}$$

Check:

$$\begin{aligned} \frac{(3)+1}{2(3)} &= \frac{2}{3} \\ \frac{4}{6} &= \frac{2}{3} \\ \frac{2}{3} &= \frac{2}{3} \text{ it's a solution!} \end{aligned}$$

9. LCD is $(2x+5)(x-4)$.

$$\begin{aligned} \left(\frac{2}{\cancel{2x+5}}\right)\cancel{(2x+5)}(x-4) &= \left(\frac{4}{\cancel{x-4}}\right)(2x+5)\cancel{(x-4)} \\ 2(x-4) &= 4(2x+5) \\ 2x-8 &= 8x+20 \\ -6x &= 28 \\ x &= \frac{28}{-6} = -\frac{14}{3} \end{aligned}$$

Check:

$$\begin{aligned} \frac{2}{2(-14/3)+5} &= \frac{4}{(-14/3)-4} \\ \frac{2}{-28/3+15/3} &= \frac{4}{(-14/6)-12/3} \\ \frac{2}{-13/3} &= \frac{4}{-26/3} \\ \frac{6}{-13} &= \frac{12}{-26} \\ \frac{6}{-13} &= \frac{6}{-13} \text{ it's a solution} \end{aligned}$$

10. LCD is $(x + 5)(3x - 2)$.

$$\left(\frac{3}{x+5}\right)\cancel{(x+5)}(3x-2) = \left(\frac{3}{3x-2}\right)(x+5)\cancel{(3x-2)}$$

$$3(3x - 2) = 3(x + 5)$$

$$3x - 2 = \frac{3}{3}(x + 5)$$

$$3x - 2 = x + 5$$

$$2x = 7$$

$$x = \frac{7}{2}$$

Check:

$$\frac{3}{(7/2) + 5} = \frac{3}{3(7/2) - 2}$$

$$\frac{3}{7/2 + 10/2} = \frac{3}{21/2 - 4/2}$$

$$\frac{3}{17/2} = \frac{3}{17/2} \text{ it's a solution}$$

11. LCD is $x + 5$.

$$(7)(x + 5) - \left(\frac{x}{x+5}\right)\cancel{(x+5)} = \left(\frac{5}{x+5}\right)\cancel{(x+5)}$$

$$7x + 35 - x = 5$$

$$6x = -30$$

$$x = -5$$

As soon as you try to check this in the original equation you will get a division by zero. Therefore $x = -5$ is not a solution. Therefore, the original equation has no solution.

12. Factor polynomials.

$$4x^2 - 1 = (2x - 1)(2x + 1) \text{ difference of squares}$$

Looking at the equation, we now see the LCD is $(2x - 1)(2x + 1)$.

$$\left(\frac{8x}{(2x-1)(2x+1)}\right)\cancel{(2x-1)}\cancel{(2x+1)} = \left(\frac{3}{2x+1}\right)(2x-1)\cancel{(2x+1)} + \left(\frac{3}{2x-1}\right)\cancel{(2x-1)}(2x+1)$$

$$8x = 3(2x - 1) + 3(2x + 1)$$

$$8x = 6x - 3 + 6x + 3$$

$$8x = 12x$$

$$-4x = 0$$

$$x = \frac{0}{-4} = 0$$

Check:

$$\frac{8(0)}{4(0)^2 - 1} = \frac{3}{2(0) + 1} + \frac{3}{2(0) - 1}$$

$$0 = 3 - 3 \text{ it's a solution}$$

13. Factor polynomials.

$$x^2 - 5x + 6 = (x - 3)(x - 2) \text{ Need two numbers whose product is 6 sum is } -5: -2, -3$$

Looking at the equation, we now see the LCD is $(x - 3)(x - 2)$.

$$\begin{aligned} \frac{6}{x-3} &= \frac{-5}{x-2} - \frac{5}{(x-3)(x-2)} \\ \left(\frac{6}{\cancel{x-3}}\right) \cancel{(x-3)}(x-2) &= \left(\frac{-5}{\cancel{x-2}}\right) (x-3)\cancel{(x-2)} - \left(\frac{5}{\cancel{(x-3)}\cancel{(x-2)}}\right) \cancel{(x-3)}\cancel{(x-2)} \\ 6(x-2) &= -5(x-3) - 5 \\ 6x - 12 &= -5x + 15 - 5 \\ 11x &= 22 \\ x &= 2 \end{aligned}$$

As soon as you try to check this in the original equation you will get a division by zero. Therefore $x = 2$ is not a solution. Therefore, the original equation has no solution.

14. $|x - 6| = 16 \Rightarrow$

$$\begin{array}{ll} x - 6 = 16 & \text{or} \quad x - 6 = -16 \\ x = 22 & \text{or} \quad x = -10 \end{array}$$

15. $|2x - 5| = 13 \Rightarrow$

$$\begin{array}{ll} 2x - 5 = 13 & \text{or} \quad 2x - 5 = -13 \\ 2x = 18 & \text{or} \quad 2x = -8 \\ x = 9 & \text{or} \quad x = -4 \end{array}$$

16. $|\frac{1}{2} - \frac{3}{4}x| + 1 = 3 \Rightarrow |\frac{1}{2} - \frac{3}{4}x| = 2 \Rightarrow$

$$\begin{array}{ll} \frac{1}{2} - \frac{3}{4}x = 2 & \text{or} \quad \frac{1}{2} - \frac{3}{4}x = -2 \\ 2 - 3x = 8 & \text{or} \quad 2 - 3x = -8 \\ -3x = 6 & \text{or} \quad -3x = -10 \\ x = -2 & \text{or} \quad x = \frac{10}{3} \end{array}$$

17. $|4 - \frac{5}{2}x| = 12 \Rightarrow$

$$\begin{array}{ll} 4 - \frac{5}{2}x = 12 & \text{or} \quad 4 - \frac{5}{2}x = -12 \\ -\frac{5}{2}x = 8 & \text{or} \quad -\frac{5}{2}x = -16 \\ x = -\frac{16}{5} & \text{or} \quad x = \frac{32}{5} \end{array}$$

18. $|x + 6| = |2x - 3| \Rightarrow$

$$\begin{array}{ll} x + 6 = 2x - 3 & \text{or} \quad x + 6 = -(2x - 3) \\ -x = -9 & \text{or} \quad x + 6 = -2x + 3 \\ x = 9 & \text{or} \quad 3x = -3 \\ x = 9 & \text{or} \quad x = -1 \end{array}$$