Questions

1. On May 18, 1990 the fastest speed of any national railroad was achieved by the French high speed train *Train á Grande Vitess* as it traveled over a distance from Cortalain to Tours, France. A commentator said that this speed was so fast that if it continued at that rate, the train would travel 6404 miles in 20 hours. How fast did the train travel on that date?

2. Two trains leave a train station at the same time. One train travels east at 50 mph. The other train travels west at 55mph. In how many hours will the two trains be 315 miles apart?

3. In warmer climates, approximate temperature predictions can be made by counting the number of chirps a cricket makes during a minute. The temperature (in Fahrenheit) decreased by 40 is equivalent to one-forth of the number of cricket chirps in a minute.

(a) Write an equation for this relationship.
(b) Approximately how many chirps per minute should be recorded if the temperature is 90 F?
(c) If a person recorded 48 cricket chirps in a minute, what would the temperature be?

4. Batman can clean up all the crime in Gotham city in 8 hrs working alone. Robin can do the same job in 12 hrs working alone. If Robin starts fighting crime at 8am and Batman joins him at 10am, then at what time will they have all the crime cleaned up?

5. A pharmacist needs to obtain a 70% alcohol solution. How many ounces of a 30% alcohol solution must be mixed with 40 ounces of an 80% alcohol solution to obtain a 70% alcohol solution?

Solutions

1. On May 18, 1990 the fastest speed of any national railroad was achieved by the French high speed train *Train á Grande Vitess* as it traveled over a distance from Cortalain to Tours, France. A commentator said that this speed was so fast that if it continued at that rate, the train would travel 6404 miles in 20 hours. How fast did the train travel on that date?

Gather Facts	Assign Variables	Basic Formula or Equation	Key Points
Train would have traveled 6404 miles in 20 hours. The distance is 6404 miles. The time is 20 hours.	We need to know the speed of the train, so let that be x.	Speed is given by the formula $x = \frac{\text{distance}}{\text{time}}$.	The speed should have units of miles per hour.

Now, we can solve the equation:

$$\begin{aligned} x &= \frac{\text{distance}}{\text{time}} \\ x &= \frac{6404\text{miles}}{20\text{hours}} \\ x &= 320.2\frac{\text{miles}}{\text{hours}} \end{aligned}$$

The speed of the train was 320.2 mph.

Check: In 20 hours, a train traveling at 320.2 miles per hour would travel a distance of $20 \times 320.2 = 6404$ miles. ✓

2. Two trains leave a train station at the same time. One train travels east at 50 mph. The other train travels west at 55mph. In how many hours will the two trains be 315 miles apart?

Gather Facts	Assign Variables	Basic Formula or Equation	Key Points
In 1 hours, EAST bound Train would have traveled 50×1 miles. In 2 hours, EAST bound Train would have traveled 50×2 miles. In t hours, EAST bound Train would have traveled $50t$ miles. In t hours, WEST bound Train would have traveled $55t$ miles.	We need to know the time it takes for something to happen, so let that be t (hours).	The distance between the two trains at time t is $50t + 55t$ miles. We want the distance to be 315 miles. Set the two expressions for the distance equal to get an equation: $50t + 55t = 315$.	

Now, we can solve the equation:

$$\begin{aligned} 50t + 55t &= 315 \\ 105t &= 315 \\ t &= 3 \end{aligned}$$

After 3 hours the trains will be 315 miles apart.

Check: After 3 hours, the EAST bound train will be $50(3) = 150$ miles from the station, and the WEST bound train will be $55(3) = 165$ miles from the station, so they will be $150 + 165 = 315$ miles apart. ✓

3. In warmer climates, approximate temperature predictions can be made by counting the number of chirps a cricket makes during a minute. The temperature (in Fahrenheit) decreased by 40 is equivalent to one-forth of the number of cricket chirps in a minute.

(a) Write an equation for this relationship.
(b) Approximately how many chirps per minute should be recorded if the temperature is 90 F?
(c) If a person recorded 48 cricket chirps in a minute, what would the temperature be?

Gather Facts	Assign Variables	Basic Formula or Equation	Key Points
	Let the number of cricket chirps in a minute be be x. Let the temperature be T (in degrees Fahrenheit).	The temperature decreased by forty is $T - 40$. One-forth the number of chirps in a minute is $\frac{1}{4}x$. These are equal: $T - 40 = \frac{1}{4}x$.	We are looking for a relationship between the number of cricket chirps and the temperature.

(a) The relationship between the number of cricket chirps per minute and the temperature is

$$T - 40 = \frac{1}{4}x$$

(b) If the temperature is 90 F, the number of chirps per minute will satisfy $90 - 40 = \frac{1}{4}x \Rightarrow 200 = x$. There should be 200 chirps per minute.

(c) If the number of chirps is 148, the temperature will satisfy $T - 40 = \frac{1}{4}(148) \Rightarrow T = 77$. The temperature is 77 F.

4. Batman cleans up 1/8 of all the crime in 1 hour.

Robin cleans up 1/12 of the all the crime in 1 hour.

Robin, from 8am to 10am working alone, cleans up $2 \times 1/12 = 1/6$ of all the crime.

At 10am, Batman joins Robin, and they work together for x hours to clean up all the crime. Working together, they clean up $1/12 + 1/8 = 5/24$ of all the crime each hour, so since they work x hours they clean up $5x/24$ of all the crime.

So, all the crime is cleaned up when $1/6 + 5x/24 = 1$. Solving for x:

$$\begin{aligned} 1/6 + 5x/24 &= 1 \\ 5x/24 &= 5/6 \\ x/24 &= 1/6 \\ x &= 4 \end{aligned}$$

So it takes $x = 4$ hours after Batman joins Robin to clean up all the crime. The crime is all cleaned up at 2pm, and they can go have a nice late lunch.

5. One jug has 40 oz of 80% alcohol solution.

A second jug has x oz of 30% alcohol solution.

Mixing them together, will create $40 + x$ oz of solution, and we want it to be 70% alcohol.

The amount of alcohol is therefore:

$$\begin{aligned} 40 \times 80\% + x \times 30\% &= (40 + x) \times 70\% \\ 32 + 0.3x &= 28 + 0.7x \\ 4 &= 0.4x \\ 10 &= x \end{aligned}$$

So it will take $x = 10$ oz of the 30% alcohol solution to get the correct mixture.