Questions

1. Simplify $-\frac{3}{2} + \sqrt{-81}$.

2. Simplify $\sqrt{-25}\sqrt{-9}$.

3. Simplify $\left(\frac{3}{4} - \frac{3}{4}i\right) + \left(\frac{9}{4} + \frac{5}{4}i\right)$.

4. Simplify $\left(\frac{1}{2} + i\right)^2$.

5. Simplify $(\sqrt{2}i)(\sqrt{6}i)$.

6. Simplify $\frac{2+i}{3-i}$.

7. Simplify $\frac{4+2i}{2-i}$.

8. Determine the real and imaginary parts of the complex number $\sqrt{i}$. *Hint:* Let $\sqrt{i} = a + bi$ $a, b \in \mathbb{R}$, and then try to determine the values of a and b.

9. Verify that $x = -2i$ is a zero of $y = x^3 - (2-i)x^2 + (2-2i)x - 4$.

10. If z is a complex number, which of the following is a real number?
(a) $z + \bar{z}$
(b) $z\bar{z}$
(c) $(z + \bar{z})^2$
(d) $(z\bar{z})^2$
(e) z^2

Solutions

1. $-\frac{3}{2}+\sqrt{-81} = -\frac{3}{2}+\sqrt{-1\cdot 9^2} = -\frac{3}{2}+\sqrt{-1}\sqrt{9^2} = -\frac{3}{2}+9i.$

2. $\sqrt{-25}\sqrt{-9} = (5i)(3i) = 15i^2 = 15(-1) = -15.$

3. $\left(\frac{3}{4}-\frac{3}{4}i\right)+\left(\frac{9}{4}+\frac{5}{4}i\right) = \left(\frac{3}{4}+\frac{9}{4}\right)+\left(-\frac{3}{4}+\frac{5}{4}\right)i = \left(\frac{3+9}{4}\right)+\left(\frac{-3+5}{4}\right)i = (3)+\left(\frac{1}{2}\right)i = 3+\frac{i}{2}.$

4. $\left(\frac{1}{2}+i\right)^2 = \left(\frac{1}{2}\right)^2+i^2+2\frac{1}{2}i = \frac{1}{4}-1+i = -\frac{3}{4}+i.$

5. $(\sqrt{2}i)(\sqrt{6}i) = \sqrt{2\cdot 6}i^2 = \sqrt{2^2\cdot 3}(-1) = \sqrt{2^2}\sqrt{3}(-1) = -2\sqrt{3}.$

6. Use the complex conjugate of denominator to divide two complex numbers.

$$\begin{aligned}\frac{2+i}{3-i} &= \frac{(2+i)(3+i)}{(3-i)(3+i)}\\ &= \frac{6+5i+i^2}{9-i^2}\\ &= \frac{6+5i-1}{9-(-1)}\\ &= \frac{5+5i}{10}\\ &= \frac{1+i}{2}\end{aligned}$$

7.

$$\begin{aligned}\frac{4+2i}{2-i} &= \frac{(4+2i)(2+i)}{(2-i)(2+i)}\\ &= \frac{8+4i+4i+2i^2}{4-i^2}\\ &= \frac{8+8i+2(-1)}{4-(-1)}\\ &= \frac{6+8i}{5}\end{aligned}$$

8. Determine the real and imaginary parts of the complex number $\sqrt{i}$. *Hint:* Let $\sqrt{i} = a+bi$ $a,b\in\mathbb{R}$, and then try to determine the values of a and b.

Let $\sqrt{i} = a + bi$, $a, b \in \mathbb{R}$.
and we need to determine a and b.

Square both sides:

$$i = (a+bi)^2$$
$$i = a^2 + 2abi + b^2 i^2 \quad \text{use } i^2 = -1.$$
$$i = a^2 + 2abi - b^2$$
$$i = a^2 - b^2 + 2abi$$

Equate real parts: $0 = a^2 - b^2$
Equate complex parts: $i = 2abi$
$\Rightarrow 1 = 2ab.$

Solve $\left.\begin{array}{l} 0 = a^2 - b^2 \\ 1 = 2ab \end{array}\right\}$ 2 equations in 2 unknowns a and b.

First equation: $a^2 = b^2$
$a = \pm b.$

Second equation ~~$b = \frac{1}{2a}$~~. $1 = 2ab$:

If $a = b$: $1 = 2(b)b \Rightarrow b^2 = \frac{1}{2} \Rightarrow b = \pm\frac{1}{\sqrt{2}}$.

so $a = \frac{1}{\sqrt{2}}$ $b = \frac{1}{\sqrt{2}}$ or $a = -\frac{1}{\sqrt{2}}$ $b = -\frac{1}{\sqrt{2}}$
are two sets of solutions.

If $a = -b$: $1 = 2(-b)b \Rightarrow b^2 = -\frac{1}{2}$. no solution since we assumed $b \in \mathbb{R}$.

Therefore $\sqrt{i} = \pm\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i\right)$.

9. Verify that $x = -2i$ is a zero of $y = x^3 - (2 - i)x^2 + (2 - 2i)x - 4$.

$$\begin{aligned}
y &= x^3 - (2 - i)x^2 + (2 - 2i)x - 4 \\
y &= (-2i)^3 - (2 - i)(-2i)^2 + (2 - 2i)(-2i) - 4 \\
&= (-2)^3 i^3 - (2 - i)(-2)^2 i^2 - 4i + 4i^2 - 4 \\
&= (-8)(i^2)i - (2 - i)(4)(-1) - 4i + 4(-1) - 4 \\
&= (-8)(-1)i - (-8 + 4i) - 4i - 8 \\
&= 8i + 8 - 4i - 4i - 8 \\
&= 0
\end{aligned}$$

10. If z is a nonreal complex number, which of the following is a real number?
(a) $z + \bar{z}$
(b) $z\bar{z}$
(c) $(z + \bar{z})^2$
(d) $(z\bar{z})^2$
(e) z^2

Let $a = a + bi$ with $a, b \in \mathbb{R}$, so $\bar{z} = a - bi$ and simplify each case.

(a) $z + \bar{z} = a + bi + a - bi = 2a \in \mathbb{R}$

(b) $z\bar{z} = (a + bi)(a - bi) = a^2 - bi^2 = a^2 + b^2 \in \mathbb{R}$

(c) $(z + \bar{z})^2 \in \mathbb{R}$ since $z + \bar{z} \in \mathbb{R}$

(d) $(z\bar{z})^2 \in \mathbb{R}$ since $z\bar{z} \in \mathbb{R}$

(e) $z^2 = (a + bi)^2 = a^2 + 2abi + b^2 i^2 = (a^2 - b^2) + 2abi$ is not $\mathbb{R}$ unless $a = 0$