Questions

Include complex solutions in your answers.

1. Solve $(x+9)^2 = 21$.

2. Solve $(4x-3)^2 = 36$.

3. Solve $(5x-2)^2 - 25 = 0$.

4. Solve by completing the square $x^2 + 6x + 2 = 0$.

5. Solve by completing the square $x^2 - 14x = -48$.

6. Solve by completing the square $\frac{x^2}{3} - \frac{x}{3} = 3$.

7. Solve by completing the square $2y^2 - y = 15$.

8. Solve $x^2 - 2x = -7$.

9. Solve $3x^2 + 8x + 3 = 2$.

Solutions

1. Use the square root property, $w^2 = a \Rightarrow w = \pm\sqrt{a}$.

$$\begin{aligned}(x+9)^2 &= 21\\ x+9 &= \pm\sqrt{21}\\ x &= -9 \pm \sqrt{21}\end{aligned}$$

2. Use the square root property.

$$\begin{aligned}(4x-3)^2 &= 36\\ 4x-3 &= \pm\sqrt{36}\\ 4x &= 3 \pm 6\\ x &= \frac{3\pm 6}{4} = \frac{3+6}{4} \text{ or } \frac{3-6}{4}\\ x &= \frac{3+6}{4} \text{ or } \frac{3-6}{4}\\ x &= \frac{9}{4} \text{ or } -\frac{3}{4}\end{aligned}$$

3. Use the square root property.

$$\begin{aligned}(5x-2)^2 - 25 &= 0\\ (5x-2)^2 &= 25\\ 5x-2 &= \pm\sqrt{25}\\ x &= \frac{2\pm 5}{5}\\ x &= \frac{2+5}{5} \text{ or } \frac{2-5}{5}\\ x &= \frac{7}{5} \text{ or } -\frac{3}{5}\end{aligned}$$

4.

$$x^2 + 6x + 2 = 0 \text{ To complete the square: } \left(\frac{6}{2}\right)^2 = 9.$$

$$x^2 + 6x + 9 - 9 + 2 = 0 \text{ underlined piece is a perfect square}$$

$$(x+3)^2 - 7 = 0$$

$$(x+3)^2 = 7$$

$$x + 3 = \pm\sqrt{7}$$

$$x = -3 \pm \sqrt{7}$$

5.

$$x^2 - 14x = -48 \text{ To complete the square: } \left(\frac{14}{2}\right)^2 = 49.$$

$$x^2 - 14x + 49 - 49 = -48 \text{ underlined piece is a perfect square}$$

$$(x-7)^2 = 1$$

$$x - 7 = \pm\sqrt{1}$$

$$x = 7 \pm 1$$

$$x = 7 + 1 \text{ or } 7 - 1$$

$$x = 8 \text{ or } 6$$

6.

$$\frac{x^2}{3} - \frac{x}{3} = 3 \text{ We MUST have a coefficient of 1 in front of the } x^2 \text{ before we complete the square.}$$

$$x^2 - x = 9$$

$$x^2 - 1x = 9 \text{ To complete the square: } \left(\frac{1}{2}\right)^2 = \frac{1}{4}.$$

$$x^2 - x + \frac{1}{4} - \frac{1}{4} = 9 \text{ underlined piece is a perfect square}$$

$$\left(x - \frac{1}{2}\right)^2 = 9 + \frac{1}{4}$$

$$\left(x - \frac{1}{2}\right)^2 = \frac{37}{4}$$

$$x - \frac{1}{2} = \pm\sqrt{\frac{37}{4}}$$

$$x = \frac{1}{2} \pm \frac{\sqrt{37}}{2}$$

7.

$$2y^2 - y = 15$$

$$y^2 - \frac{1}{2}y = \frac{15}{2} \quad \text{To complete the square: } \left(\frac{1}{4}\right)^2 = \frac{1}{16}.$$

$$y^2 - \frac{1}{2}y + \frac{1}{16} - \frac{1}{16} = \frac{15}{2}$$

$$\left(y - \frac{1}{4}\right)^2 - \frac{1}{16} = \frac{15}{2}$$

$$\left(y - \frac{1}{4}\right)^2 = \frac{1}{16} + \frac{120}{16}$$

$$y - \frac{1}{4} = \pm\sqrt{\frac{121}{16}}$$

$$y = \frac{1}{4} \pm \frac{11}{4}$$

$$y = \frac{1}{4} + \frac{11}{4} \text{ or } \frac{1}{4} - \frac{11}{4}$$

$$y = 3 \text{ or } -\frac{5}{2}$$

8. Solve by completing the square.

$$x^2 - 2x = -7 \quad \text{To complete the square: } \left(\frac{2}{2}\right)^2 = 1.$$

$$x^2 - 2x + 1 - 1 = -7$$

$$(x-1)^2 - 1 = -7$$

$$x - 1 = \pm\sqrt{-6}$$

$$x - 1 = \pm\sqrt{6}i$$

$$x = 1 \pm \sqrt{6}i$$

9. Solve by completing the square.

$$3x^2 + 8x + 3 = 2$$

$$x^2 + \frac{8}{3}x + 1 = \frac{2}{3}$$

$$x^2 + \frac{8}{3}x = -\frac{1}{3} \quad \text{To complete the square: } \left(\frac{1}{2} \cdot \frac{8}{3}\right)^2 = \frac{16}{9}.$$

$$x^2 + \frac{8}{3}x + \frac{16}{9} - \frac{16}{9} = -\frac{1}{3}$$

$$\left(x + \frac{4}{3}\right)^2 = \frac{16}{9} - \frac{1}{3}$$

$$\left(x + \frac{4}{3}\right)^2 = \frac{13}{9}$$

$$x + \frac{4}{3} = \pm\sqrt{\frac{13}{9}}$$

$$x = -\frac{4}{3} \pm \frac{\sqrt{13}}{3}$$