Questions

1. Explain how the graphs of $y = f(x) = \sqrt{x}$ and $y = f(-x)$ are related.

2. Explain how the graphs of $y = f(x) = \sqrt{x}$ and $y = -f(x)$ are related.

3. Explain how the graphs of $y = f(x) = x^3$ and $y = f(x) - 3$ are related.

4. Explain how the graphs of $y = f(x) = 1/x$ and $y = f(x - 2)$ are related.

5. Explain how the graphs of $y = f(x) = x$ and $y = 2f(x)$ are related.

6. Explain how the graphs of $y = f(x) = x$ and $y = f(3x)$ are related.

7. Sketch $y = \dfrac{12}{x - 7}$. State the domain and range, and determine intervals of increasing, decreasing, constant.

8. Sketch $y = -\dfrac{3}{x + 2} - 1$. State the domain and range, and determine intervals of increasing, decreasing, constant.

9. Sketch $y = -4|2x| + 1$. State the domain and range, and determine intervals of increasing, decreasing, constant.

10. Sketch $y = -3\sqrt{x - 2} - 1$. State the domain and range, and determine intervals of increasing, decreasing, constant.

11. Determine, using algebra, if the function $f(x) = \dfrac{12x}{1 - x^2}$ is odd, even, or neither odd nor even.

12. Determine, using algebra, if the function $f(x) = x^4 + x$ is odd, even, or neither odd nor even.

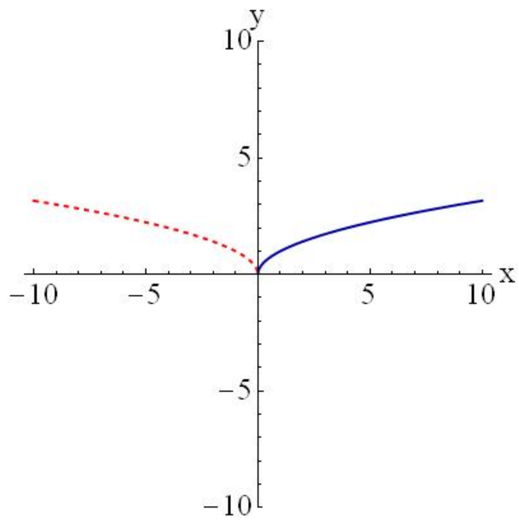
Solutions

My solutions will include sketches using *Mathematica* so you can best see how the two functions are related. You should be able to draw these sketches by hand.

1. Explain how the graphs of $y = f(x) = \sqrt{x}$ and $y = f(-x)$ are related.

The graph of $y = \sqrt{x}$ is a square root function (blue).

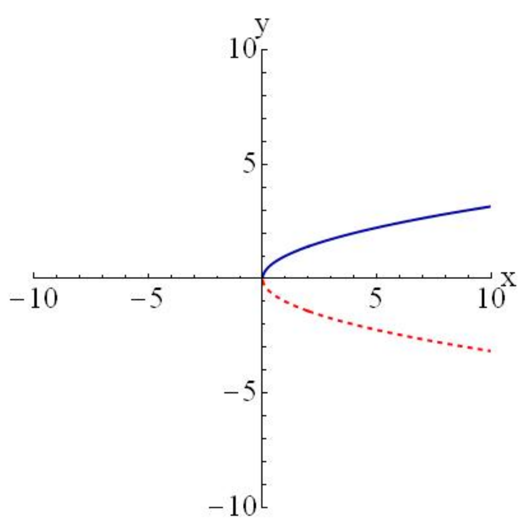
The graph of $y = f(-x)$ is modified inside the f (so it is a horizontal change) and since it is $f(-x)$ this is a reflection about the y-axis (red).



2. Explain how the graphs of $y = f(x) = \sqrt{x}$ and $y = -f(x)$ are related.

The graph of $y = \sqrt{x}$ is a square root function (blue).

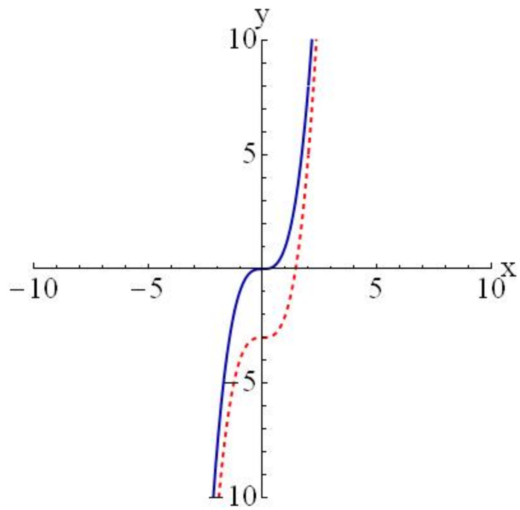
The graph of $y = -f(x)$ is modified outside the f (so it is a vertical change) and since it is $-f(x)$ this is a reflection about the x-axis (red).



3. Explain how the graphs of $y = f(x) = x^3$ and $y = f(x) - 3$ are related.

The graph of $y = x^3$ is a cube function (blue).

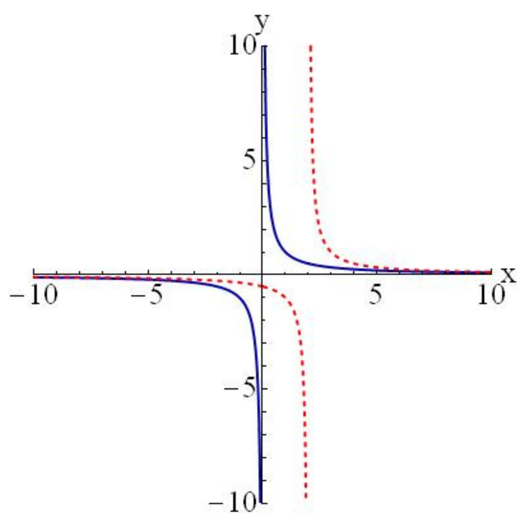
The graph of $y = f(x) - 3$ is modified outside the f (so it is a vertical change) and since it is $f(x) - 3$ this is shifted down three units (red).



4. Explain how the graphs of $y = f(x) = 1/x$ and $y = f(x - 2)$ are related.

The graph of $y = 1/x$ is a reciprocal function (blue).

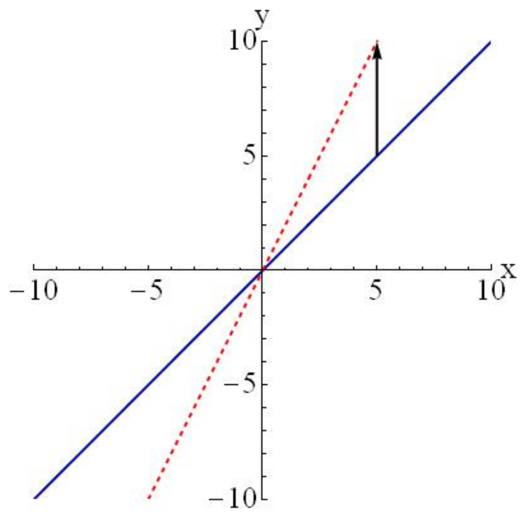
The graph of $y = f(x - 2)$ is modified inside the f (so it is a horizontal change) and since it is $f(x - 2)$ this is shifted to the right two units (red).



5. Explain how the graphs of $y = f(x) = x$ and $y = 2f(x)$ are related.

The graph of $y = x$ is a linear function (blue).

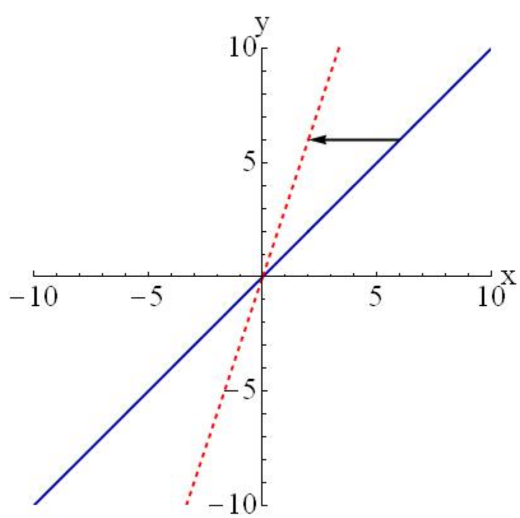
The graph of $y = 2f(x)$ is modified outside the f (so it is a vertical change) and since it is $2f(x)$ this is stretched vertically by two units (red). The arrow shows the vertical stretch of the point $(5, 5)$ to the point $(5, 10)$ on the new graph.



6. Explain how the graphs of $y = f(x) = x$ and $y = f(3x)$ are related.

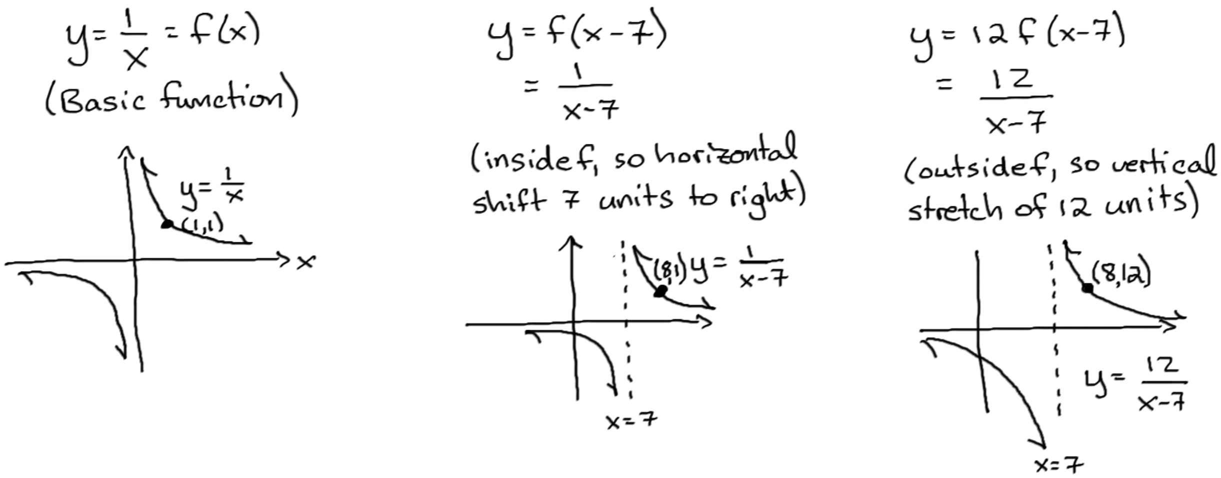
The graph of $y = x$ is a linear function (blue).

The graph of $y = f(3x)$ is modified inside the f (so it is a horizontal change) and since it is $f(3x)$ this is compressed horizontally by three units (red). The arrow shows the horizontal compression of the point $(6, 6)$ to the point $(2, 6)$ on the new graph.



7. Sketch $y = \dfrac{12}{x-7}$. State the domain and range, and determine intervals of increasing, decreasing, constant.

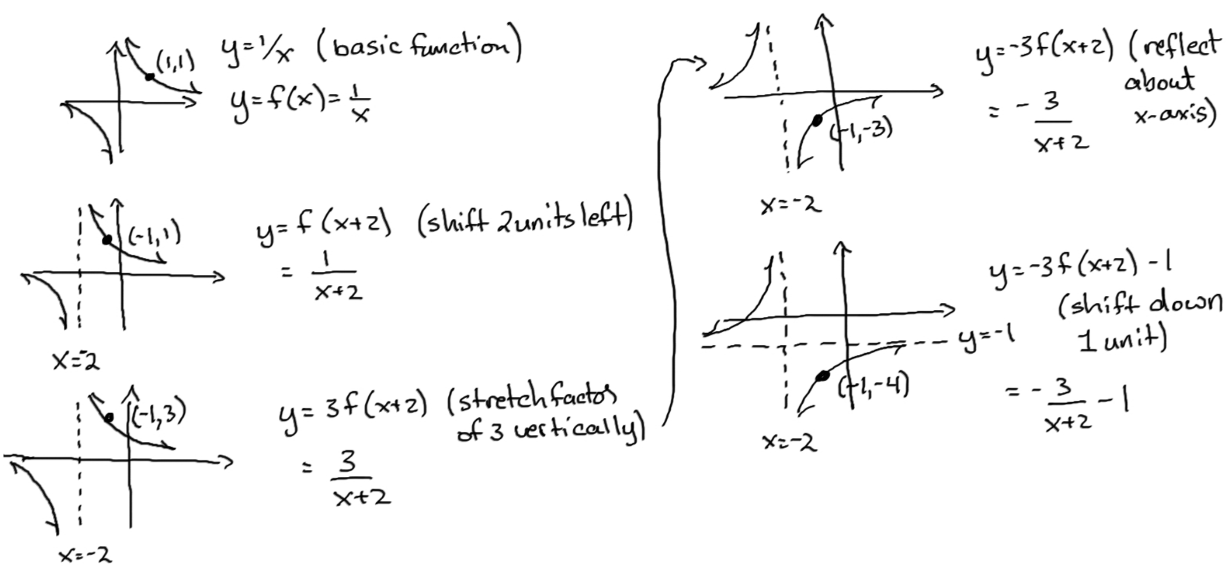
I am tracking what happens to one point as we make the transformations.



From the sketch, the domain is $x \in (-\infty, 7) \cup (7, \infty)$ and the range is $y \in (-\infty, 0) \cup (0, \infty)$.

The function is decreasing for $x \in (-\infty, 7)$ and $x \in (7, \infty)$.

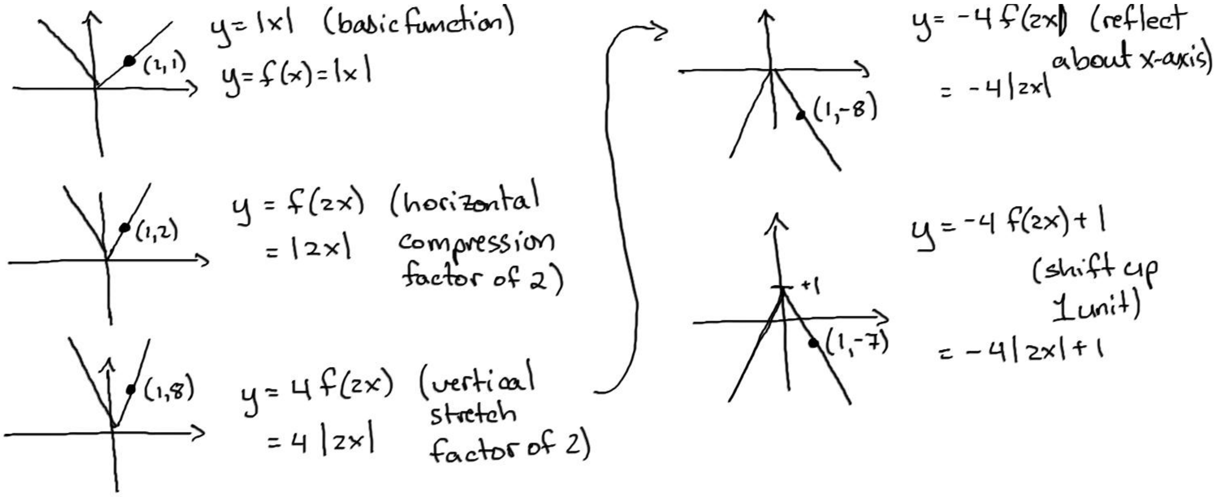
8. Sketch $y = -\dfrac{3}{x+2} - 1$. State the domain and range, and determine intervals of increasing, decreasing, constant.



From the sketch, the domain is $x \in (-\infty, -2) \cup (-2, \infty)$ and the range is $y \in (-\infty, -1) \cup (-1, \infty)$.

The function is increasing for $x \in (-\infty, -2)$ and $x \in (-2, \infty)$.

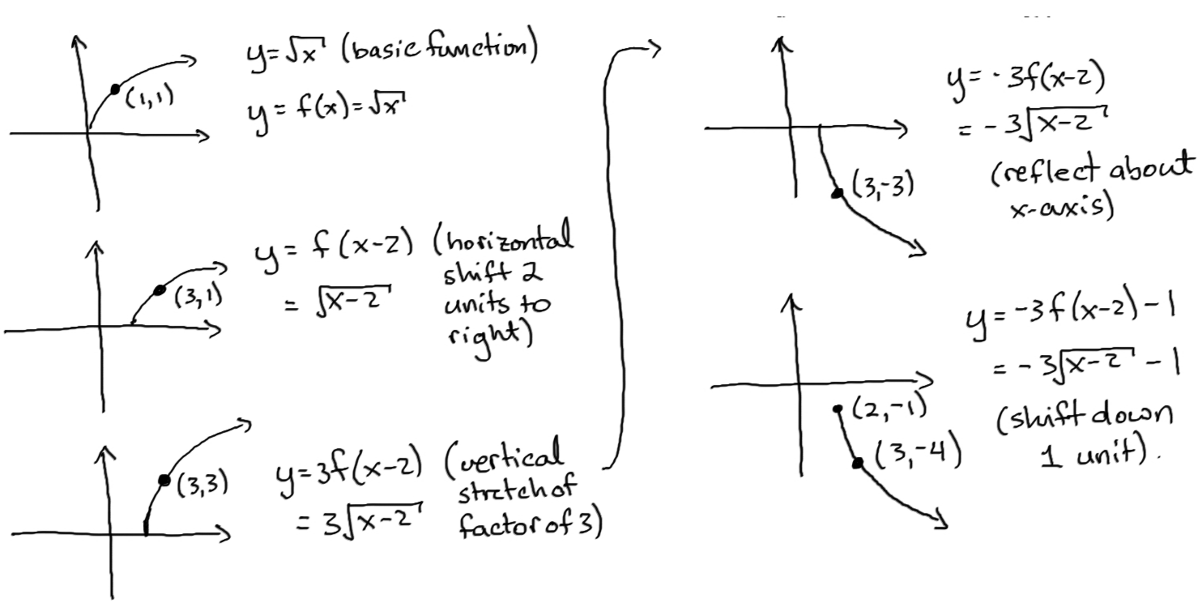
9. Sketch $y = -4|2x| + 1$. State the domain and range, and determine intervals of increasing, decreasing, constant.



From the sketch, the domain is $x \in (-\infty, \infty)$ and the range is $y \in (-\infty, 1]$.

The function is increasing for $x \in (-\infty, 0)$ and decreasing for $x \in (0, \infty)$.

10. Sketch $y = -3\sqrt{x-2} - 1$. State the domain and range, and determine intervals of increasing, decreasing, constant.



From the sketch, the domain is $x \in (2, \infty)$ and the range is $y \in (-\infty, -1]$.

The function is decreasing for $x \in (2, \infty)$.

11. We must work out $f(-x)$ and see if it equals $f(x)$ (even), $-f(x)$ (odd), or neither (neither)!

$$\begin{aligned} f(-x) &= \frac{12(-x)}{1-(-x)^2} \\ &= \frac{-12x}{1-x^2} \\ &= -\frac{12x}{1-x^2} \\ &= -f(x) \end{aligned}$$

So $f(x) = \dfrac{12}{1-x^3}$ odd.

12. $f(-x) = (-x)^4 + (-x) = x^4 - x$. So $f(x) = \dfrac{12}{1-x^3}$ neither even nor odd.