Questions

1. Given that $f(x) = 3x + 17$ and $g(x) = \dfrac{1}{1+x}$, compute $(f \circ g)(x)$ and $(g \circ f)(x)$. Do not worry about the domains.

2. If $f(x) = \sqrt{x-1}$ and $g(x) = x^4$, find the functions fg, $f - g$, $f + g$, f/g and each of their domains.

3. Find $(f \circ g)(3)$ and $(g \circ f)(-2)$ when $f(x) = x^2 - 1$ and $g(x) = 2x - 3$.

4. Find $f(g(x))$ and $g(f(x))$ if $f(x) = 1/(x-1)$ and $g(x) = \sqrt{x}$. State the domain of each.

5. Decompose the function $f(x) = \dfrac{1}{\cos(x^3)}$ so that each decomposed function is a more basic functions.

Solutions

1. Given that $f(x) = 3x + 17$ and $g(x) = \dfrac{1}{1+x}$, compute $(f \circ g)(x)$ and $(g \circ f)(x)$. Do not worry about the domains.

$$\begin{aligned}
(f \circ g)(x) &= f(g(x)) \\
&= f(\frac{1}{1+x}) \\
&= 3(\frac{1}{1+x}) + 17 \\
&= \frac{3}{1+x} + 17 \\
(g \circ f)(x) &= g(f(x)) \\
&= g(3x + 17) \\
&= \frac{1}{1 + (3x + 17)} = \frac{1}{3x + 18}
\end{aligned}$$

2. If $f(x) = \sqrt{x-1}$ and $g(x) = x^4$, find the functions $(fg)(x)$, $(f - g)(x)$, $(f + g)(x)$, $(f/g)(x)$ and each of their domains.

Domain of f is $x \in [1, \infty) = A$. Domain of g is $x \in (-\infty, \infty) = B$. Intersection $A \cap B = [1, \infty)$.

$$\begin{aligned}
(fg)(x) &= f(x)g(x) \\
&= \sqrt{x-1} \cdot x^4 \\
&= x^4\sqrt{x-1}, \text{ domain } [1, \infty) \\
(f - g)(x) &= f(x) - g(x) \\
&= \sqrt{x-1} - x^4, \text{ domain } [1, \infty) \\
(f + g)(x) &= f(x) + g(x) \\
&= \sqrt{x-1} + x^4, \text{ domain } [1, \infty) \\
(f/g)(x) &= f(x)/g(x) \\
&= \frac{\sqrt{x-1}}{x^4}, \text{ domain } [1, 0) \cup (0, \infty)
\end{aligned}$$

3. Find $(f \circ g)(3)$ and $(g \circ f)(-2)$ when $f(x) = x^2 - 1$ and $g(x) = 2x - 3$.

$$\begin{aligned}
(f \circ g)(3) &= f(g(3)) \\
&= f(2(3) - 3) \\
&= f(3) \\
&= (3)^2 - 1 \\
&= 8
\end{aligned}$$

$$\begin{aligned}
(g \circ f)(-2) &= g(f(-2)) \\
&= g((-2)^2 - 1) \\
&= g(3) \\
&= 2(3) - 3 \\
&= 3
\end{aligned}$$

4. Find $f(g(x))$ and $g(f(x))$ if $f(x) = 1/(x-1)$ and $g(x) = \sqrt{x}$. State the domain of each.

$$\begin{aligned}
f(g(x)) &= f(\sqrt{x}) \\
&= \frac{1}{\sqrt{x} - 1}
\end{aligned}$$

The domain of $f(g(x))$ is $x \in [0, 1) \cup (1, \infty)$.

$$\begin{aligned}
g(f(x)) &= g\left(\frac{1}{x-1}\right) \\
&= \sqrt{\frac{1}{x-1}} \\
&= \frac{1}{\sqrt{x-1}}
\end{aligned}$$

The domain of $g(f(x))$ is $x \in (1, \infty)$.

5. Decompose the function $f(x) = \dfrac{1}{\cos x^3}$ so that each decomposed function is a more basic functions.

Two evaluate this, you would cube, then take the cosine, then take the reciprocal. This gives you the decomposition. Pick $z(x) = x^3$, $h(x) = \cos x$, $g(x) = \frac{1}{x}$. Check:

$$f(x) = g(h(z(x))) = g(h(x^3)) = g(\cos x^3) = \frac{1}{\cos x^3}.$$