

You solutions may follow different paths than mine but still be correct.

### Questions

1. Write down the rules of exponents.

2. Simplify  $\left(\frac{3xy^{-2}}{y^3}\right)^{-2}$

4. Simplify  $2a^{-1/6}b^{3/4}$  so there are no negative exponents.

5. Simplify  $-5y^{-2/3}$  so there are no negative exponents.

6. Simplify  $(27)^{2/3}$

7. Simplify  $(-27)^{5/3}$

8. Simplify  $(-64)^{2/3}$

9. Simplify  $(x^{-1/3}y^{2/3})(x^{1/3}y^{1/4})$

10. Factor out the common factor  $2a$  in  $10a^{5/4} - 4a^{8/5}$

11. Simplify  $\sqrt[3]{-125x^{30}}$ .

12. Simplify  $\sqrt[3]{-27a^6}$ .

13. Simplify  $(3\sqrt{5} + \sqrt{3})(\sqrt{2} + 2\sqrt{5})$ .

14. Rationalize denominator in  $\frac{\sqrt{3x} - 2\sqrt{y}}{\sqrt{3x} + \sqrt{y}}$ .

15. Rationalize numerator in  $\frac{\sqrt{3x} - 2\sqrt{y}}{\sqrt{3x} + \sqrt{y}}$ .

## Solutions

1. The rules of exponents are:

- $x^0 = 1$  if  $x \neq 0$  ( $0^0$  is indeterminate and is dealt with in calculus).
- Product Rule:  $x^a \cdot x^b = x^{a+b}$ .
- Quotient Rule:  $\frac{x^a}{x^b} = x^{a-b}$ .
- Power Rule:  $(x^a)^b = x^{ab}$ .
- Product Raised to Power Rule:  $(xy)^a = x^a y^a$ .
- Quotient Raised to a Power Rule:  $\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}$  if  $y \neq 0$ .
- Negative Exponent:  $x^{-n} = \frac{1}{x^n}$ , if  $x \neq 0$ .

2.

$$\begin{aligned} \left(\frac{3xy^{-2}}{y^3}\right)^{-2} &= \frac{(3)^{-2}(x)^{-2}(y^{-2})^{-2}}{(y^3)^{-2}} \text{ Using Power Rule} \\ &= \frac{y^4}{3^2 x^2 (y^{-6})} \text{ Simplify, using Power Rule and Negative Exponent Rule} \\ &= \frac{y^4 y^6}{9x^2} \text{ Simplify, using Negative Exponent Rule} \\ &= \frac{y^{4+6}}{9x^2} \text{ Simplify, using Product Rule} \\ &= \frac{y^{10}}{9x^2} \text{ Simplify} \end{aligned}$$

4.  $2a^{-1/6}b^{3/4} = \frac{2b^{3/4}}{a^{1/6}}$ .

5.  $-5y^{-2/3} = \frac{-5}{y^{2/3}}$ .

6.  $(27)^{2/3} = (3^3)^{2/3} = (3)^2 = 9$ .

7.  $(-27)^{5/3} = ((-3)^3)^{5/3} = (-3)^5 = -243$ .

8.  $(-64)^{2/3} = ((-4)^3)^{2/3} = (-4)^2 = 16$ .

9.  $(x^{-1/3}y^{2/3})(x^{1/3}y^{1/4}) = x^{-1/3+1/3}y^{2/3+1/4} = x^0y^{8/12+3/12} = y^{11/12}$

10.  $10a^{5/4} - 4a^{8/5} = 2a \cdot 5a^{1/4} - 2a \cdot 2a^{3/5} = 2a(5a^{1/4} - 2a^{3/5})$ .

11.

$$\begin{aligned} \sqrt[3]{-125x^{30}} &= (-125x^{30})^{1/3} \\ &= (-125)^{1/3}(x^{30})^{1/3} \\ &= ((-5)^3)^{1/3}x^{10} \\ &= (-5)x^{10} \\ &= -5x^{10} \end{aligned}$$

12.

$$\begin{aligned}\sqrt[3]{-27a^6} &= (-27a^6)^{1/3} \\ &= (-27)^{1/3} (a^6)^{1/3} \\ &= ((-3)^3)^{1/3} a^2 \\ &= (-3)a^2 = -3a^2\end{aligned}$$

13.

$$\begin{aligned}(3\sqrt{5} + \sqrt{3})(\sqrt{2} + 2\sqrt{5}) &= (3\sqrt{5})(\sqrt{2} + 2\sqrt{5}) + (\sqrt{3})(\sqrt{2} + 2\sqrt{5}) \\ &= (3\sqrt{5})(\sqrt{2}) + (3\sqrt{5})(2\sqrt{5}) + (\sqrt{3})(\sqrt{2}) + (\sqrt{3})(2\sqrt{5}) \\ &= 3\sqrt{5 \cdot 2} + (6)(5) + \sqrt{3 \cdot 2} + 2\sqrt{3 \cdot 5} \\ &= 3\sqrt{10} + 30 + \sqrt{6} + 2\sqrt{15}\end{aligned}$$

14.

$$\begin{aligned}\frac{\sqrt{3x} - 2\sqrt{y}}{\sqrt{3x} + \sqrt{y}} &= \frac{(\sqrt{3x} - 2\sqrt{y})(\sqrt{3x} - \sqrt{y})}{(\sqrt{3x} + \sqrt{y})(\sqrt{3x} - \sqrt{y})} \\ &= \frac{3x - 2\sqrt{3x}\sqrt{y} - \sqrt{3x}\sqrt{y} + 2(y)}{3x - y} \\ &= \frac{3x - 3\sqrt{3xy} + 2y}{3x - y}\end{aligned}$$

15.

$$\begin{aligned}\frac{\sqrt{3x} - 2\sqrt{y}}{\sqrt{3x} + \sqrt{y}} &= \frac{(\sqrt{3x} - 2\sqrt{y})(\sqrt{3x} + 2\sqrt{y})}{(\sqrt{3x} + \sqrt{y})(\sqrt{3x} + 2\sqrt{y})} \\ &= \frac{3x - 4y}{3x + 2\sqrt{3x}\sqrt{y} + \sqrt{3x}\sqrt{y} + 2y} \\ &= \frac{3x - 4y}{3x + 3\sqrt{3xy} + 2y}\end{aligned}$$