

Questions

1. Multiply $\frac{1}{2}(2x + 3x^2 + 5x^3)$.
2. Multiply $(-4x^3 + 6x^2 - 5x)(-7xy^2)$.
3. Multiply $(5y + 1)(6y - 5)$.
4. What is wrong with this multiplication: $(x - 2)(-3) = 3x - 6$?
5. What is wrong with this multiplication: $-(3x - 7) = -3x - 7$?
6. Multiply $\left(\frac{1}{3}x + \frac{1}{5}\right)\left(\frac{1}{3}x - \frac{1}{2}\right)$.
7. Multiply $\left(5x - \frac{1}{5}\right)\left(5x + \frac{1}{5}\right)$.
8. Multiply $(6x + 5)^2$.
9. Multiply $(8x - 3)^2$.
10. Multiply $\left(\frac{3}{5}x - \frac{1}{3}\right)\left(\frac{3}{5}x + \frac{1}{3}\right)$.
11. Divide $\frac{12x^2 + 19x + 5}{3x + 1}$.
12. Divide $\frac{4x^3 + 4x^2 - 19x - 15}{2x + 5}$.
13. Divide $\frac{9y^3 - 30y^2 + 31y - 4}{3y - 5}$.

Solutions

1. $\frac{1}{2}(2x + 3x^2 + 5x^3) = \frac{1}{2}(2x) + \frac{1}{2}(3x^2) + \frac{1}{2}(5x^3) = x + \frac{3}{2}x^2 + \frac{5}{2}x^3$

2.

$$\begin{aligned} (-4x^3 + 6x^2 - 5x)(-7xy^2) &= (-4x^3)(-7xy^2) + (6x^2)(-7xy^2) - (5x)(-7xy^2) \text{ distribute} \\ &= 28x^{3+1}y^2 - 42x^{2+1}y^2 + 35x^{1+1}y^2 \text{ product rule of exponents} \\ &= 28x^4y^2 - 42x^3y^2 + 35x^2y^2 \text{ simplify} \end{aligned}$$

3.

$$\begin{aligned} (5y+1)(6y-5) &= (5y+1)(6y) + (5y+1)(-5) && \text{distribute} \\ &= 5y(6y) + 1(6y) + 5y(-5) + 1(-5) && \text{distribute} \\ &= 30y^2 + 6y - 25y - 5 && \text{simplify} \\ &= 30y^2 - 19y - 5 && \text{collect like terms} \end{aligned}$$

4. Let's do the multiplication and then compare to find the error.

$$\begin{aligned} (x - 2)(-3) &= (x)(-3) - (2)(-3) \\ &= -3x - (-6) = -3x + 6 \end{aligned}$$

Signs are incorrect.

5. Let's do the multiplication and then compare to find the error.

$$\begin{aligned} -(3x - 7) &= -1(3x - 7) \\ &= -1(3x) + (-1)(-7) \\ &= -3x + 7 \end{aligned}$$

The minus sign was not distributed to the last term.

6.

$$\begin{aligned} \left(\frac{1}{3}x + \frac{1}{5}\right)\left(\frac{1}{3}x - \frac{1}{2}\right) &= \left(\frac{1}{3}x + \frac{1}{5}\right)\left(\frac{1}{3}x\right) + \left(\frac{1}{3}x + \frac{1}{5}\right)\left(-\frac{1}{2}\right) \text{ distribute} \\ &= \left(\frac{1}{3}x\right)\left(\frac{1}{3}x\right) + \left(\frac{1}{5}\right)\left(\frac{1}{3}x\right) + \left(\frac{1}{3}x\right)\left(-\frac{1}{2}\right) + \left(\frac{1}{5}\right)\left(-\frac{1}{2}\right) \text{ distribute} \\ &= \left(\frac{1}{9}x^2\right) + \left(\frac{1}{15}x\right) + \left(-\frac{1}{6}x\right) + \left(-\frac{1}{10}\right) \text{ simplify} \\ &= \frac{1}{9}x^2 + \frac{2}{30}x - \frac{5}{30}x - \frac{1}{10} \text{ collect like terms: add fractions} \\ &= \frac{1}{9}x^2 - \frac{3}{30}x - \frac{1}{10} \text{ collect like terms: add fractions} \\ &= \frac{1}{9}x^2 - \frac{1}{10}x - \frac{1}{10} \text{ simplify} \end{aligned}$$

7. Difference of squares, $(a + b)(a - b) = a^2 - b^2$.

$$\left(5x - \frac{1}{5}\right)\left(5x + \frac{1}{5}\right) = 25x^2 - \frac{1}{25}$$

8. A binomial squared with addition, $(a + b)^2 = a^2 + 2ab + b^2$.

$$(6x + 5)^2 = 36x^2 + 60x + 25$$

9. A binomial squared with subtraction, $(a - b)^2 = a^2 - 2ab + b^2$.

$$(8x - 3)^2 = 64x^2 - 48x + 9$$

10. Difference of squares, $(a + b)(a - b) = a^2 - b^2$.

$$\left(\frac{3}{5}x - \frac{1}{3}\right)\left(\frac{3}{5}x + \frac{1}{3}\right) = \frac{9}{25}x^2 - \frac{1}{9}$$

11.

$$\begin{array}{r} 4x+5 \\ 3x+1 \overline{) 12x^2+19x+5} \\ \underline{12x^2+4x} \\ 15x+5 \\ \underline{15x+5} \\ 0 \text{ remainder.} \end{array} \quad \text{so } \frac{12x^2+19x+5}{3x+1} = 4x+5$$

check: $(4x+5)(3x+1) = (4x+5)(3x) + (4x+5)(1)$
 $= (4x)(3x) + (5)(3x) + (4x)(1) + (5)(1)$
 $= 12x^2 + 15x + 4x + 5$
 $= 12x^2 + 19x + 5 \quad \text{OK!}$

12.

$$\begin{array}{r} 2x^2-3x-2 \\ 2x+5 \overline{) 4x^3+4x^2-19x-15} \\ \underline{4x^3+10x^2} \\ -6x^2-19x-15 \\ \underline{-6x^2-15x} \\ -4x-15 \\ \underline{-4x-10} \\ -5 \text{ remainder.} \end{array}$$

Therefore, $\frac{4x^3+4x^2-19x-15}{2x+5} = 2x^2-3x-2 + \left(\frac{-5}{2x+5}\right)$

Check: $(2x^2-3x-2)(2x+5) - 5$ (do you know why we do this for the check?)
 $= (2x^2)(2x+5) + (-3x)(2x+5) + (-2)(2x+5) - 5$
 $= (2x^2)(2x) + (2x^2)(5) + (-3x)(2x) + (-3x)(5) + (-2)(2x) + (-2)(5) - 5$
 $= 4x^3 + 10x^2 - 6x^2 - 15x - 4x - 10 - 5$
 $= 4x^3 + 4x^2 - 19x - 15 \quad \text{OK!}$

13.

$$\begin{array}{r}
 \overline{) 9y^3 - 30y^2 + 31y - 4} \\
 \underline{9y^3 - 15y^2} \\
 -15y^2 + 31y - 4 \\
 \underline{-15y^2 + 25y} \\
 6y - 4 \\
 \underline{6y - 10} \\
 6 \text{ remainder.}
 \end{array}$$

Therefore, $\frac{9y^3 - 30y^2 + 31y - 4}{3y - 5} = 3y^2 - 5y + 2 + \frac{6}{3y - 5}$