1| Since we cannot take the square root of a negative number (and get a real number), the domain of f is all x such that $x-12 \geq 0$ or $x \in [12,\infty)$.

2| The $\sqrt{x}$ requires $x \in [0,\infty)$

The $\ln x$ requires $x \in (0,\infty)$

We exclude $x=1$ since $\ln 1 = 0$ would lead to division by zero.

Domain $(0,1) \cup (1,\infty)$

3| $y = 4e^{3x-9}$

interchange x and y: $x = 4e^{3y-9}$

solve for y: $\frac{x}{4} = e^{3y-9}$

$\ln\left(\frac{x}{4}\right) = \ln e^{3y-9}$

$\ln(x/4) = 3y-9$

$y = f^{-1}(x) = \frac{\ln(x/4)+9}{3}$

check: $f(f^{-1}(x)) = f\left(\frac{\ln(x/4)+9}{3}\right)$

$= 4\exp\left(\cancel{3}\left(\frac{\ln(x/4)+9}{\cancel{3}}\right)-9\right)$

$= 4\exp(\ln(x/4)+9-9)$

$= 4\exp(\ln(x/4))$

$= 4(x/4) = x.$ ✓

4| slope $= \frac{\Delta y}{\Delta x} = \frac{-7-(-11)}{-3-5} = -\frac{1}{2}$

$y = mx+b$

$y = -\frac{1}{2}x + b$

$-7 = -\frac{1}{2}(-3)+b$ (use one point to determine b)

$b = -7 - \frac{3}{2} = -\frac{17}{2}$

so $y = -\frac{1}{2}x - \frac{17}{2}$.

5| $(f\circ f)(x) = f(f(x))$

$= f(x^2-4)$

$= (x^2-4)^2-4$

$= x^4-8x^2+16-4$

$= x^4-8x^2+12$

$(g\circ f)(x) = g(f(x))$

$= g(x^2-4)$

$= \sqrt{x^2-4}+4$.

6| $f(x) = x^2-4x+5$

$= \underbrace{x^2-4x+4}-4+5$

$= (x-2)^2+1$

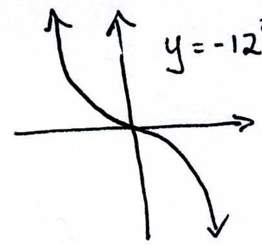
7| $f(x) = -(12x-7)^2(34x+89)^3$

Degree 5.

Zeros: $x = 7/12$ multiplicity 2 so f does not changes sign.

$x = -89/34$ multiplicity 3 so f changes sign, and since multiplicity is greater than 1 f will be horizontal at the zero.

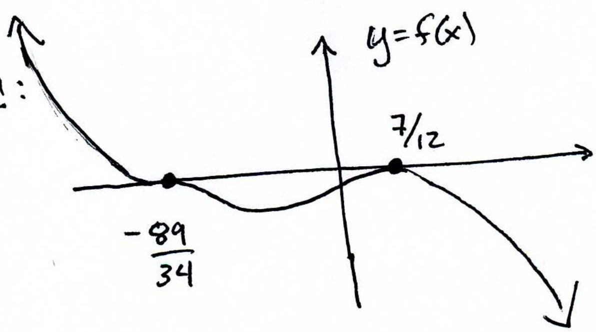
End behaviour: If $|x|$ large, $f(x) \sim -(12x)^2(34x)^3 = -12^2 \cdot 34^3 x^5$



$\lim_{x \to \infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = \infty$.

sketch:



8| $f(x) = \dfrac{2(x-1)}{(x+1)(x-3)}$

Zeros: $x = 1$ multiplicity 1, f changes sign.

vertical asymptotes: $\left.\begin{array}{l} x=-1 \\ x=3 \end{array}\right\}$ multiplicity 1, f changes sign.

End behaviour: If $|x|$ large, $f(x) \sim \dfrac{2(x)}{(x)(x)} = \dfrac{2}{x}$. So if $x \to \infty$, $f(x) > 0$ (positive)

sign chart:

$f(x)$	$-$	∞	$+$	0	$-$	∞	$+$	
x		-1		1		3		

So $f(x) \leq 0$ if $x \in (-\infty, -1) \cup [1, 3)$.

9) $\dfrac{\Delta y}{\Delta x} = \dfrac{f(x_0+h)-f(x_0)}{h}$

$f(x) = ax^2+bx+c$

$f(x_0) = ax_0^2+bx_0+c$

$f(x_0+h) = a(x_0+h)^2+b(x_0+h)+c$
$\quad = ax_0^2+2ax_0h+ah^2+bx_0+bh+c$

$$= \frac{\cancel{ax_0^2}+2ax_0h+ah^2+\cancel{bx_0}+bh+\cancel{c}-\cancel{ax_0^2}-\cancel{bx_0}-\cancel{c}}{h}$$

$$= \frac{2ax_0h+ah^2+bh}{h} = \frac{\cancel{h}(2ax_0+ah+b)}{\cancel{h}} = 2ax_0+ah+b$$

10) $f(x) = \dfrac{(3-x)(3+x)^2}{(12-4x)^2} = \dfrac{(3-x)(3+x)^2}{4^2(3-x)^2} = \dfrac{(3+x)^2}{16(3-x)}$, $x-3 \neq 0$

(no hole because we are going to have a vertical asymptote) at $x=3$

Zeros: $x=-3$ multiplicity 2 so f does not change sign.

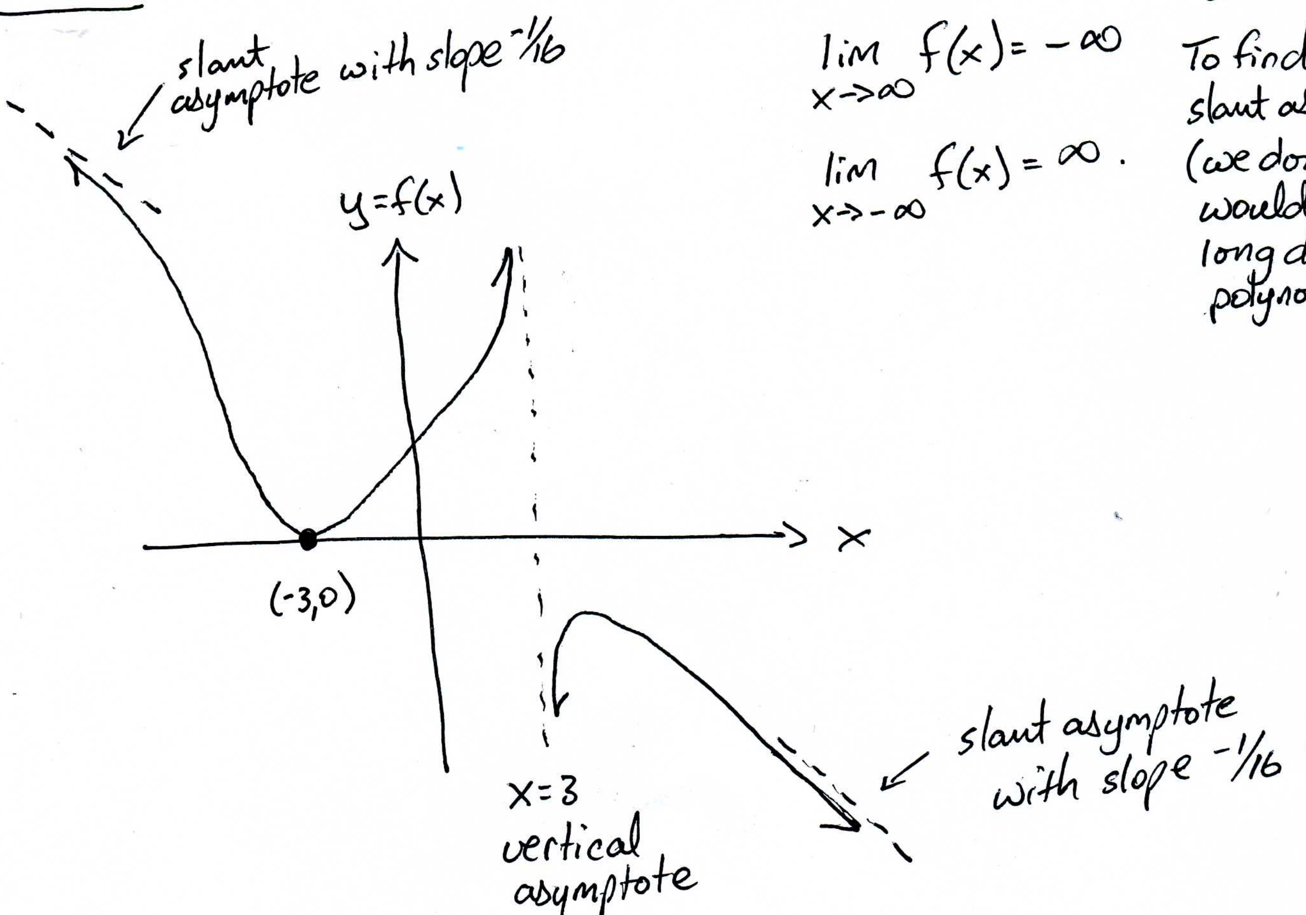
vertical asymptote: $x=3$ multiplicity 1 so f changes sign.

End Behaviour: If $|x|$ large, $f(x) \approx \dfrac{(-x)(x)^2}{(-4x)^2} = -\dfrac{x}{4^2} = -\dfrac{x}{16}$ so there will be a slant asymptote with slope $-1/16$.

$\lim_{x\to\infty} f(x) = -\infty$

$\lim_{x\to-\infty} f(x) = \infty$.

To find actual slant asymptote (we don't have y-int) would require long division of polynomials.



11) $f(3)=0$ means $x-3$ is a factor. Use long division to factor it out.

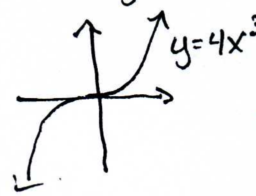
$$\begin{array}{r} 4x^2-4x+1 \\ x-3\,\overline{)\,4x^3-16x^2+13x-3} \\ \underline{4x^3-12x^2} \\ -4x^2+13x \\ \underline{-4x^2+12x} \\ x-3 \\ \underline{x-3} \\ 0 \end{array}$$

Also, $4x^2-4x+1 = (2x-1)^2$

so $f(x) = (x-3)(2x-1)^2$

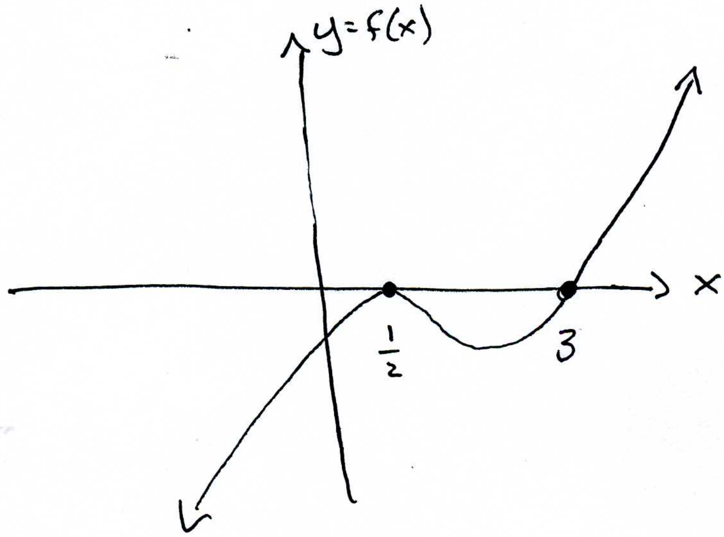
Zeros: $x=3$ multiplicity 1, f changes sign.
$x=\frac{1}{2}$ multiplicity 2, f does not change sign.

end behaviour: if $|x|$ large, $f(x) \sim (x)(2x)^2 = 4x^3$.

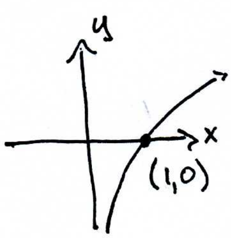


$\lim_{x\to\infty} f(x) = \infty$

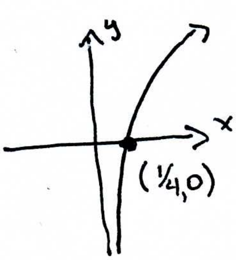
$\lim_{x\to-\infty} f(x) = -\infty$



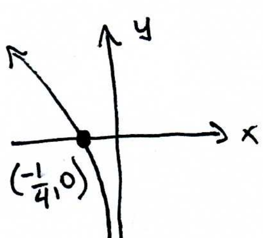
12)



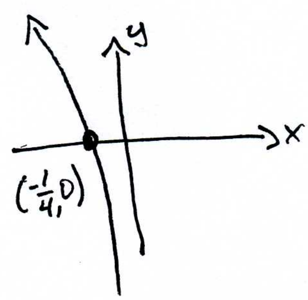
$y = \ln x$
$= f(x)$



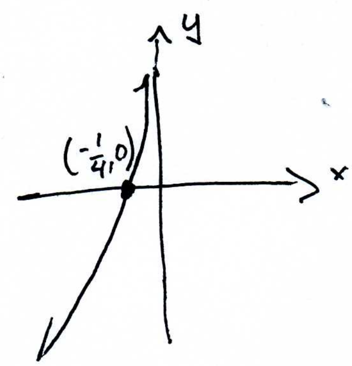
$y = f(4x)$
$= \ln(4x)$
horizontal shrink by factor of 4.



$y = f(-4x)$
$= \ln(-4x)$
reflect about y-axis.



$y = 3f(-4x)$
$= 3\ln(-4x)$
vertical stretch by factor of 4.



$y = -3f(-4x)$
$= -3\ln(-4x)$
reflect about x-axis.

13 $\ln(xy) + 2\ln(yz^2) - \ln(xz) = \ln(xy) + \ln(y^2z^4) - \ln(xz)$

$$= \ln(xy \cdot y^2z^4) - \ln(xz)$$

$$= \ln(xy^3z^4) - \ln(xz)$$

$$= \ln\left(\frac{xy^3z^4}{xz}\right)$$

$$= \ln(y^3z^3)$$

14

$$\frac{44}{1+4e^{-x/7}} = 32$$

$$44 = 32 + 128e^{-x/7}$$

$$12 = 128e^{-x/7}$$

$$\frac{12}{128} = e^{-x/7}$$

$$e^{-x/7} = \frac{3}{32}$$

$$\ln e^{-x/7} = \ln(3/32)$$

$$-\frac{x}{7} = \ln(3/32)$$

$$x = -7\ln(3/32) = 7\ln(32/3)$$

15

$$\frac{(x+1)^2}{2^2} + \frac{(y-1)^2}{1^2} = 1$$

center $(-1,1)$

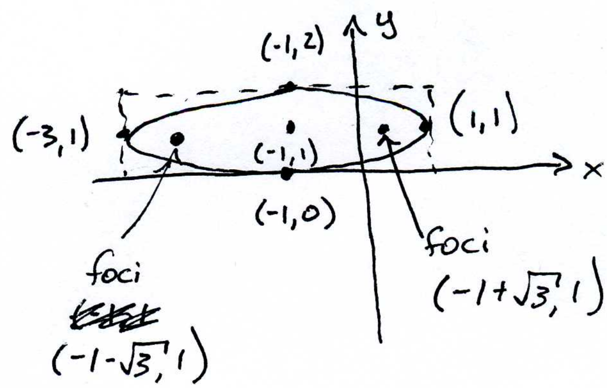
Box x width $= 2a = 4$

y width $= 2b = 2$

$c = \sqrt{4-1} = \sqrt{3}$

foci ~~$(-1, 1+\sqrt{3})$~~ $(-1-\sqrt{3}, 1)$

~~$(-1, 1-\sqrt{3})$~~ $(-1+\sqrt{3}, 1)$



vertices

$(1,1)$

$(-3,1)$

16 $\ln(x) - \frac{1}{2}\ln(x+4) = 0$

$$\ln(x) - \ln[(x+4)^{1/2}] = 0$$

$$\ln\left(\frac{x}{(x+4)^{1/2}}\right) = 0$$

$$e^{\ln\left(\frac{x}{(x+4)^{1/2}}\right)} = e^0$$

$$\frac{x}{(x+4)^{1/2}} = 1$$

$$x = (x+4)^{1/2}$$

$$x^2 = x+4$$

$$x^2 - x - 4 = 0$$

$$x = \frac{1 \pm \sqrt{1-4(1)(-4)}}{2}$$

$$= \frac{1 \pm \sqrt{17}}{2}$$

From original equation $\left.\begin{array}{l} x>0 \\ x+4>0 \end{array}\right\}$ both satisfied if $x>0$.

Only one solution $x = \frac{1+\sqrt{17}}{2}$. $\frac{1-\sqrt{17}}{2}$ is extraneous.

17
$$(g\circ f)(x) = g(f(x))$$
$$= g\left(\tfrac{1}{2}\ln(x+2)\right)$$
$$= g\left(\ln\sqrt{x+2}\right)$$
$$= e^{\ln\sqrt{x+2}}$$
$$= \sqrt{x+2}$$

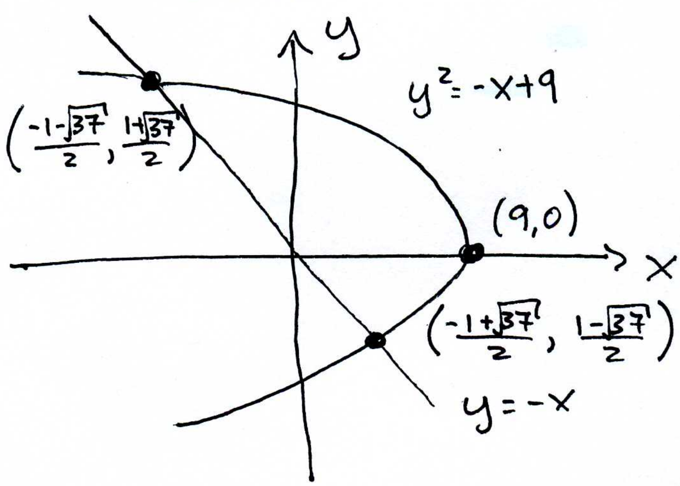
18
$$y^2 = -x+9$$
$$y = -x$$

Sub second into first:
$$(-x)^2 = -x+9$$
$$x^2+x-9=0$$
$$x = \frac{-1\pm\sqrt{1-4(1)(-9)}}{2}$$
$$= \frac{-1\pm\sqrt{37}}{2}$$

if $x = \frac{-1+\sqrt{37}}{2}$, $y=-x$
$$= \frac{1-\sqrt{37}}{2}$$
$$\left(\frac{-1+\sqrt{37}}{2}, \frac{1-\sqrt{37}}{2}\right)$$

if $x = \frac{-1-\sqrt{37}}{2}$, $y = \frac{1+\sqrt{37}}{2}$
$$\left(\frac{-1-\sqrt{37}}{2}, \frac{1+\sqrt{37}}{2}\right)$$

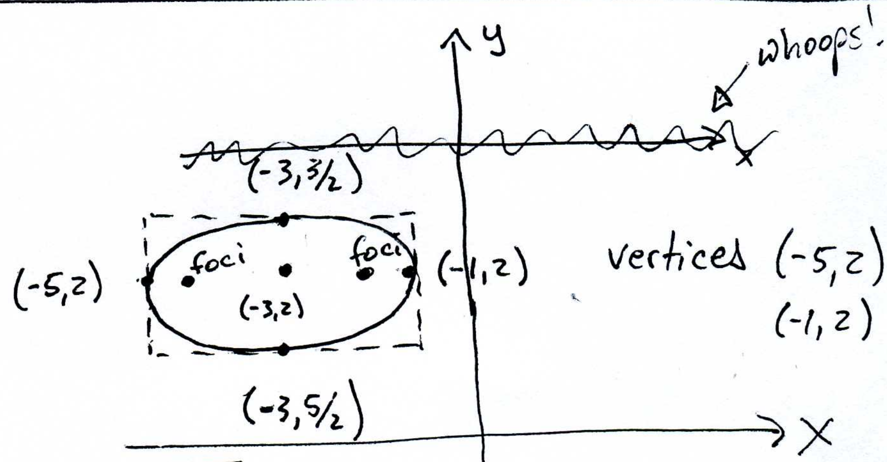
$$y^2 = -x+9$$
$$y^2 = -(x-9)$$
parabola opens to left
$$4p=-1 \rightarrow p = -1/4.$$
~~center~~ vertex $(9,0)$



19
$$(x+3)^2+16(y-2)^2=4$$
$$\frac{(x+3)^2}{2^2} + \frac{(y-2)^2}{(\frac{1}{2})^2} = 1$$

Center $(-3,2)$.

Box width in x is $2a=4$
width in y is $2b=1$



vertices $(-5,2)$
$(-1,2)$

$$c = \sqrt{4-\frac{1}{4}} = \frac{\sqrt{15}}{2}$$

foci $\left(-3+\frac{\sqrt{15}}{2}, 2\right)$
$\left(-3-\frac{\sqrt{15}}{2}, 2\right)$.

20|

$$y^2 = x$$
$$x^2 = -8y$$

Sub 1^{st} into 2^{ND}:

$$(y^2)^2 = -8y$$
$$y^4 = -8y$$
$$y(y^3+8) = 0$$

$y=0$ or $y^3+8=0$

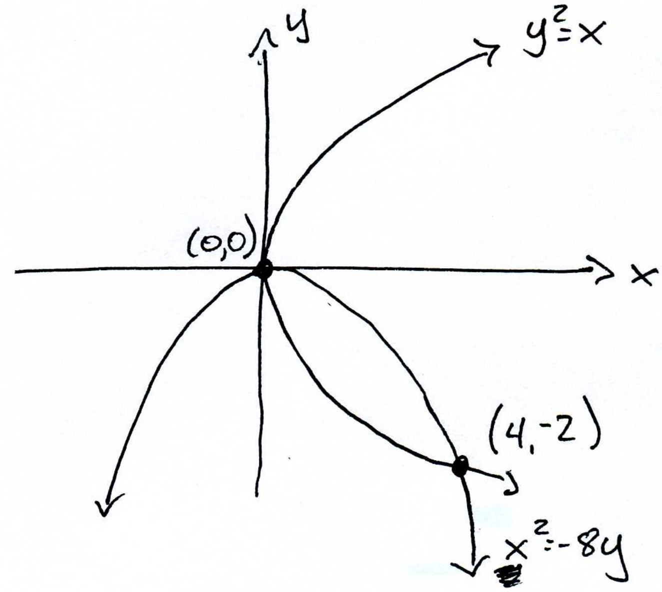
$$y = (-8)^{1/3} = -2.$$

For $y=0$, $x=y^2=0$: $(0,0)$

$y=-2$, $x=(-2)^2=4$: $(4,-2)$

$y^2=x$ is parabola, opens to right, Center $(0,0)$.

$x^2=-8y$ is parabola, opens down, center $(0,0)$.



21|

$$f(x) = \frac{1}{\sqrt{x}}$$
$$f(x+h) = \frac{1}{\sqrt{x+h}}$$

$$\frac{\Delta y}{\Delta x} = \frac{f(x+h)-f(x)}{h}$$
$$= \frac{1}{h}\left[\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right]$$
$$= \frac{1}{h}\left[\frac{\sqrt{x}-\sqrt{x+h}}{\sqrt{x+h}\sqrt{x}}\right]\left(\frac{\sqrt{x}+\sqrt{x+h}}{\sqrt{x}+\sqrt{x+h}}\right) \quad \text{rationalize}$$
$$= \frac{1}{h}\,\frac{x-(x+h)}{\sqrt{x+h}\sqrt{x}\left(\sqrt{x}+\sqrt{x+h}\right)}$$
$$= \frac{-1}{\sqrt{x+h}\sqrt{x}\left(\sqrt{x}+\sqrt{x+h}\right)}$$

22

$$y = \frac{2x+1}{4-5x}$$

interchange x & y: $\quad x = \dfrac{2y+1}{4-5y}$

solve for y:

$$(4-5y)x = 2y+1$$

$$4x - 5yx = 2y+1$$

$$4x-1 = 2y+5yx$$

$$4x-1 = y(2+5x)$$

$$y = f^{-1}(x) = \frac{4x-1}{2+5x}$$

$$f(f^{-1}(x)) = f\left(\frac{4x-1}{2+5x}\right)$$

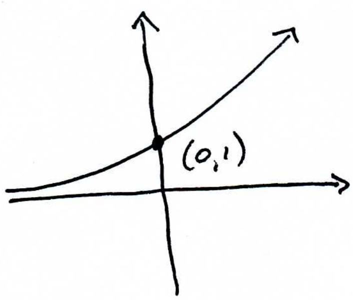
$$= \frac{2\left(\frac{4x-1}{2+5x}\right)+1}{4-5\left(\frac{4x-1}{2+5x}\right)}$$

$$= \frac{\left(\frac{8x-2+2+5x}{2+5x}\right)}{\left(\frac{8+20x-20x+5}{2+5x}\right)}$$

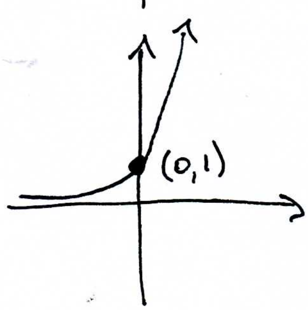
$$= \frac{13x}{2+5x} \cdot \frac{2+5x}{13}$$

$$= x \checkmark$$

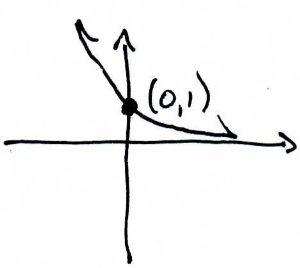
23



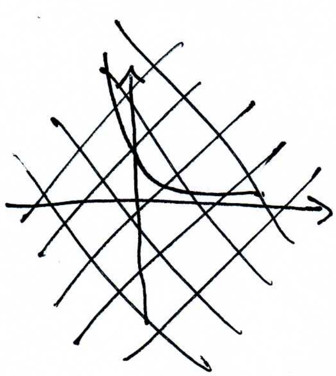
$y = e^x$
$= f(x)$



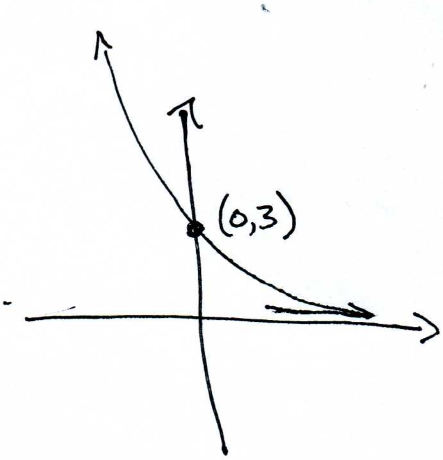
$y = f(4x)$
$= e^{4x}$
horizontal squeeze by factor of 4.



$y = f(-4x)$
$= e^{-4x}$
reflect across y-axis.



$y = 3f(-4x)$
$= 3e^{-4x}$
vertical stretch by factor of 3.



24) $f(3)=0$ means $x-3$ is a factor. Use long division to divide it out.

$$
\begin{array}{r|l}
 & 3x^2-2x-4 \\
\hline
x-3 & 3x^3-11x^2+2x+12 \\
 & 3x^3-9x^2 \\
\hline
 & -2x^2+2x \\
 & -2x^2+6x \\
\hline
 & -4x+12 \\
 & -4x+12 \\
\hline
 & 0
\end{array}
$$

Also, $3x^2-2x-4$ can be factored.

$$x = \frac{2 \pm \sqrt{4-4(3)(-4)}}{6}$$

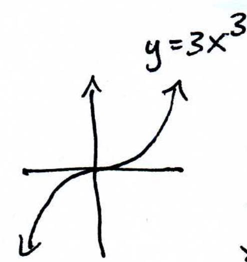
$$= \frac{2 \pm \sqrt{52}}{6}.$$

A bit yucky, but these are real numbers. Note to get correct leading coefficient

$$f(x) = 3(x-3)\left(x - \left(\frac{2+\sqrt{52}}{6}\right)\right)\left(x - \left(\frac{2-\sqrt{52}}{6}\right)\right)$$

Zeros: $x=3$, $x = \frac{2\pm\sqrt{52}}{6}$ $\Big\}$ multiplicity 1, so f changes sign.

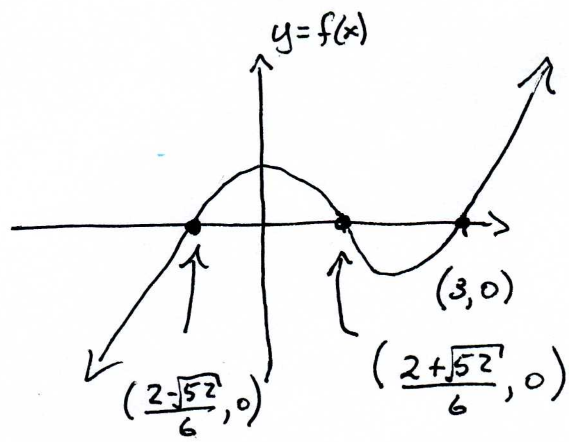
End behaviour: If $|x|$ is large, $f(x) \sim 3x^3$.



$$\lim_{x\to\infty} f(x) = \infty$$

$$\lim_{x\to-\infty} f(x) = -\infty$$

sketch:



25| $f(x) = \dfrac{(x-1)(x-2)^2}{(x-3)(x-4)}$

zeros: $x=1$ multiplicity 1 so f changes sign
$x=2$ multiplicity 2 so f does not change sign.

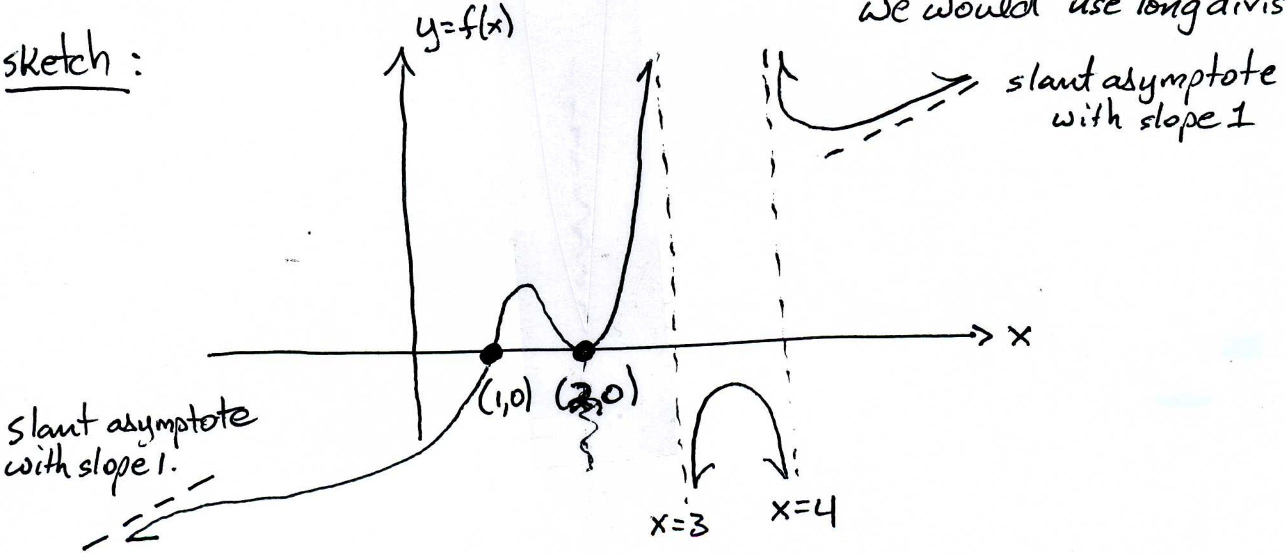
vertical asymptotes: $\left.\begin{array}{l} x=3 \\ x=4 \end{array}\right\}$ multiplicity 1, f changes sign.

End Behaviour: If $|x|$ large, $f(x) \sim \dfrac{x \cdot x^2}{x \cdot x} = x$

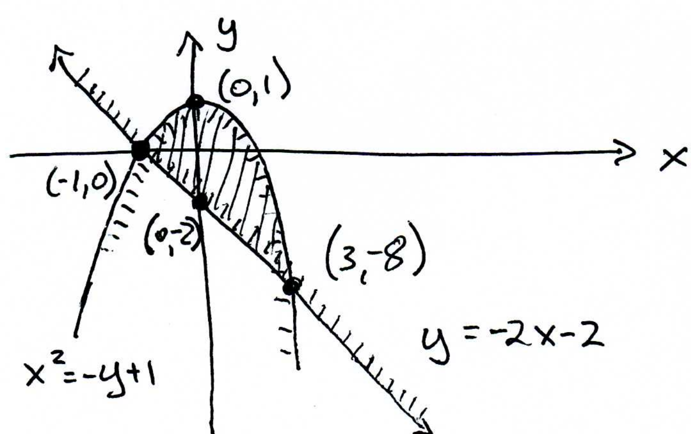
$\lim_{x\to\infty} f(x) = \infty$
$\lim_{x\to -\infty} f(x) = -\infty$

slant asymptote with slope 1.
If we wanted y-int of slant asymptote, we would use long division of polynomials.

sketch :



26| $x^2 \leq -y+1$
$y \geq -2x-2$



$x^2 \leq -y+1$

$x^2 = -y+1$
$x^2 = -(y-1)$
parabola, opens down
vertex (0,1)
If $y=0$, $x=\pm 1$. → (1,0)
(-1,0)
Test point (0,0) : $0 \leq 0+1$ True!

$y \geq -2x-2$

$y = -2x-2$
straight line.
If $x=0$, $y=-2$: (0,-2)
If $y=0$, $x=-1$: (-1,0)
Test Point (0,0) : $0 \geq 0-2$ True

Intersection $x^2 = -y+1$
$y = -2x-2$

Put 2ND into 1st: $x^2 = -(-2x-2)+1$
$x^2 - 2x - 3 = 0$
$(x+1)(x-3) = 0$ → $x=-1$, $x=3$

If $x=-1$, $y=-2(-1)-2=0$ → (-1,0)
If $x=3$, $y=-2(3)-2=-8$ → (3,-8)

27| $P(t)$ is number of bunnies after t days.

$P(0) = P_0 = 123$

$P(24) = 2 \cdot 123$

$P(48) = 2^2 \cdot 123$

$P(72) = 2^3 \cdot 123$

$P(t) = 2^{t/24} \cdot 123$ is number of rabbits after t days.

$1000 = 123 \; 2^{t/24}$

$\ln\left(\frac{1000}{123}\right) = \ln\left(2^{t/24}\right)$

$\ln\left(\frac{1000}{123}\right) = t \frac{\ln(2)}{24}$

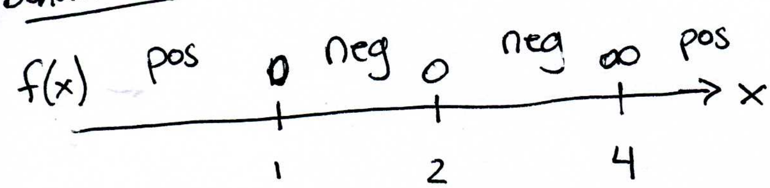
$\Rightarrow t = \frac{24 \ln\left(\frac{1000}{123}\right)}{\ln 2}$ days is when you have 1000 rabbits.

28| $f(x) = \frac{(x-1)(x-2)^2}{(x-4)}$

Zeros: $x=1$ multiplicity 1, f changes sign.
$x=2$ multiplicity 2, f does not change sign.

Vertical Asymptotes: $x=4$ multiplicity 1, f changes signs.

End Behaviour: If $|x|$ is large $f(x) \sim \frac{x \cdot x^2}{x} = x^2$ so $\lim_{x\to\infty} f(x) = \infty$ is positive.
$\lim_{x\to-\infty} f(x) = +\infty$ is positive.



So $f(x) \le 0$ if $x \in [1,4)$

29| ~~$y^2 - 6y + 9 + 8x - 8 = 0$~~

$y^2 - 6y + 8x + 1 = 0$

$\underbrace{y^2 - 6y + 9}$ $- 9 + 8x + 1 = 0$

$(y-3)^2 = -8x + 8$

$(y-3)^2 = -8(x-1)$

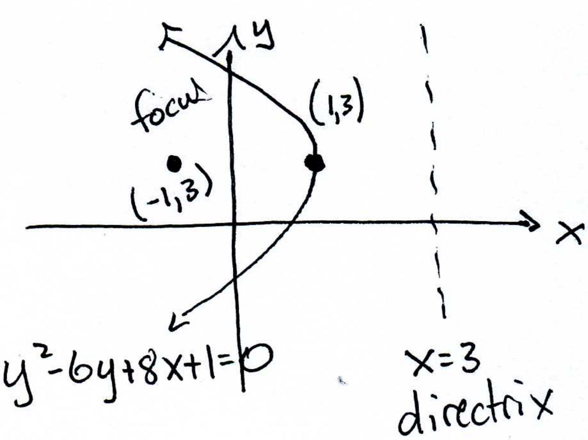
parabola opens left

vertex ~~Center~~ $(1,3)$

$+8 = 4p \Rightarrow p = +2$.

focus $(1-2, 3) = (-1, 3)$

directrix is $x = 1 + p = 1 + 2 = 3$



30. $4x^2 - 8x + 54y - 9y^2 = 41$

$$4x^2 - 8x - 9y^2 + 54y = 41$$

$$4(\underbrace{x^2 - 2x + 1}-1) - 9(\underbrace{y^2 - 6y + 9}-9) = 41$$

$$4(x-1)^2 - 4 - 9(y-3)^2 + 81 = 41$$

$$4(x-1)^2 - 9(y-3)^2 = -36$$

$$9(y-3)^2 - 4(x-1)^2 = 36$$

$$\frac{(y-3)^2}{2^2} - \frac{(x-1)^2}{3^2} = 1 \qquad \frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

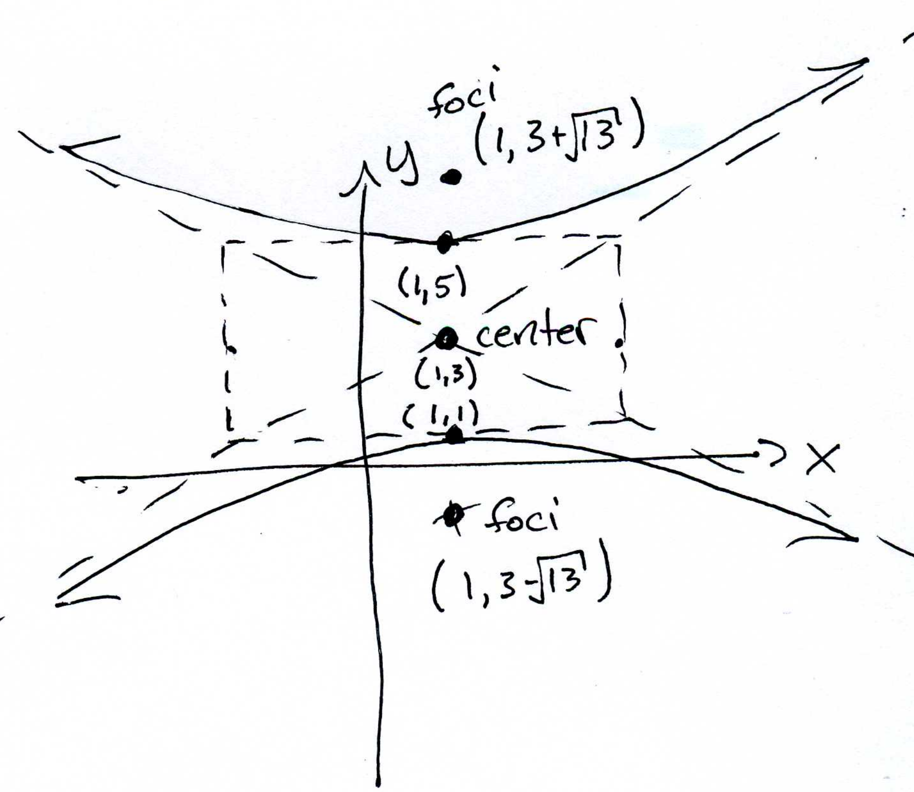
Hyperbola.

Center $(1,3) = (h,k)$

Box width in x is $2a = 6$

width in y is $2b = 4$

$$c = \sqrt{a^2+b^2} = \sqrt{9+4} = \sqrt{13}$$



If $x=1$, then

$$\frac{(y-3)^2}{2^2} = 1$$

$$y - 3 = \pm 2$$

$$y = 5, 1$$

$(1,1)$ $(1,5)$ vertices.

foci $(1, 3+c) = (1, 3+\sqrt{13})$

$(1, 3-c) = (1, 3-\sqrt{13})$