Final Exam Preparation

The final exam will be 10 questions long, some with two parts. Material for the final can be drawn from anything we have studied.

Suggestions to guide your preparation:

- Start with the final exam review questions to get an overview of the material.

  Attempt the questions before reading my solution, if you get stuck, read only as much of the solution as you need to get unstuck, and then try to finish the problem. Refer to the text as necessary.

  This will provide you with an overview of the material you need to be studying.

- Then review Practice Problems and previous tests.

- Then review the concepts described in the concept reviews for each test on the course webpage.

  Can you talk about the concepts? Do you know the basic results from the concept review? For example, do you know the logarithm and exponential laws? What is the algebraic way of checking if a function is odd or even? Ask yourself these questions, and make sure you can answer them.

- Make notes on the topics you are studying.

  Write short sentences to describe how to solve problems. Include example problems if a certain type of problem appears frequently.

- Then do problems from the text for which you have solutions, that are similar to the problems you have seen so far in your test preparation.

- Branch out and do other types of problems that appeared less frequently throughout the course.

- Studying in many short sessions is more effective than one or two marathon studying sessions.

  Consider making a time schedule which maps out when and what you will study, to help you organize and prepare for all your finals.

  You might choose a long term time frame (Friday Morning: History, Friday Afternoon: Precalculus, etc), and a short term time frame for each day that lists what exactly you will focus on. The short term time frame can be created every day and be more flexible.

  Create goals which you can reasonably be expected to meet.

- Get as much sleep as possible while you study for finals. Come to your final exams well rested, and mentally sharp.

- Study in an environment that mimics the environment the test will take place in. It should be quiet and clear of clutter.

Once you have begun your review, here are some things you might do.

- For a given chapter (or section), create practice "tests" for yourself, maybe three or four questions which you have the solution to, and then answer them without reference to the text. Correct your test, or have a friend correct your test and you correct theirs. Do not move on to other questions until you have mastered these ones. You might consider imposing a time limit on these mini-tests.

- If you do study in groups, also study alone so you can focus on the types of questions you need to work on.

- Before the final, practice some mini-tests which draw from all the chapters we have studied.

- Talk with me if there are questions you have.

- Algebra
  - solving linear equations
  - completing the square
  - the quadratic formula
  - interval notation, set notation, number line notation
- Sketching Linear Equations
  - slope $m = \dfrac{\Delta y}{\Delta x}$
  - parallel lines have same slope, but different y-intercept
  - perpendicular lines have slopes whose product is -1
  - equations of straight lines, $y = mx + b$, $y - y_1 = m(x - x_1)$
- Sketching Quadratic Equations of Form $y = f(x) = ax^2 + bx + c$
  - opens up $(a > 0)$ or down $(a < 0)$
  - quadratic formula for zeros: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
  - finding vertex $x = -b/(2a)$, $y = f(-b/(2a))$ (memorized formula), or
  - finding vertex (h, k) by completing the square and comparing to the vertex form $y = a(x - h)^2 + k$.
- Functions
  - domain/range
  - vertical asymptotes
  - horizontal asymptotes
  - increasing/decreasing
  - even/odd/neither (algebraic and graphical)
  - end behaviour
- Properties of the 12 Basic Functions
- Average Rate of Change of function f over interval $[a, a + h]$ is $\dfrac{\Delta y}{\Delta x} = \dfrac{f(a + h) - f(a)}{h}$
- Inverse Functions
  - algebraic technique to find inverse function
  - graphical technique to find inverse function
  - cancelation equations
- Algebra of Functions
  - algebraic combinations of functions, such as fg, $f + g$, $f - g$, and f/g and their domains
  - composition of functions (no domain for composition on this test)
- Simple Graphical Transformation of Function $f(x)$, with $c > 1$
  - shift up: $f(x) + c$
  - shift down: $f(x) - c$
  - shift left: $f(x + c)$
  - shift right: $f(x - c)$
  - reflect about x-axis: $-f(x)$
  - reflect about y-axis: $f(-x)$
  - stretch vertically: $cf(x)$
  - compress horizontally: $f(cx)$
- Power Functions $f(x) = kx^a, a \in \mathbb{R}, a \neq 0$
  - square root function
  - direct variation
  - inverse variation
- Monomial Functions $f(x) = kx^n, n = 0, 1, 2, 3, \ldots$
  - end behaviour for n even, n odd
  - sketching monomials
  - reciprocal function

- Polynomials
  - terminology: term, coefficients, leading term
  - local extrema
  - end behaviour: $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$
  - zeros of polynomials, multiplicity, crossing x-axis
- Factoring
  - difference of squares: $a^2 - b^2 = (a-b)(a+b)$
  - perfect square: $a^2 \pm 2ab + b^2 = (a \pm b)^2$
  - difference of cubes: $a^3 - b^3 = (a-b)(a^2+ab+b^2)$
  - sum of cubes: $a^3 + b^3 = (a+b)(a^2-ab+b^2)$
  - factoring by using quadratic formula
  - factoring by using long division algorithm
- Complex Zeros
  - arithmetic of complex numbers
  - complex conjugate of $a+bi$ is $a-bi$
  - a non-real zero of f is not an x-intercept of f
  - a polynomial of odd degree will have at least one real zero
- Zeros of Polynomials
  - long division algorithm for polynomials
  - remainder theorem
  - factor theorem
  - rational zero theorem
- Sketching a Polynomial
  - examine end behaviour (horizontal asymptotes, slant asymptotes)
  - find any x-intercepts (factor the polynomial) including multiplicity
  - know how to treat irreducible factors (irreducible means there are no real valued factors)
  - find the y-intercept, which is $f(0)$ (it might be a point of interest)
- Sketching a Rational Function
  - examine end behaviour (horizontal asymptotes, slant asymptotes)
  - look for vertical asymptotes (factor the denominator) including multiplicity
  - find any x-intercepts (factor the numerator) including multiplicity
  - know how to treat irreducible factors (irreducible means there are no real valued factors)
  - find the y-intercept, which is $f(0)$ (it might be a point of interest)
- Solving Equalities
  - solving polynomial equations $f(x) = 0$
  - solving rational equations $f(x)/g(x) = 0$
    - lowest common denominator
    - extraneous solutions
    - indeterminant forms ($\frac{0}{0}$ is an indeterminant form, you need to do some work to determine what it is)
- Solving Inequalities
  - sign chart
  - polynomial inequalities
  - rational inequalities
  - radical inequalities, absolute value inequalities

- exponential functions $f(x) = e^{kx}$ or $f(x) = b^x$
  - properties of exponents: If x and y are real numbers, and $b > 0$ is real, then
    1. $b^x \cdot b^y = b^{x+y}$
    2. $\frac{b^x}{b^y} = b^{x-y}$
    3. $(b^x)^y = b^{xy}$
  - nomenclature: base, exponent
  - exponential growth (b^x with $b > 1$) and exponential decay (b^x with $0 < b < 1$)
  - the natural base $e \sim 2.71828\ldots$
  - exponential growth (e^{kx} with $k > 0$) and exponential decay (e^{kx} with $k < 0$)
  - the basic function $f(x) = e^x$
  - sketching and graphical transformations of exponential functions
- constructing exponential population models (looking for the pattern)
- logistic functions $y = \frac{c}{1 + ae^{-kx}}$
- logarithmic functions $f(x) = \ln x$ (natural logarithm) and $f(x) = \log_b x$
  - interpreted as the inverse function of the exponential function
    1. $\ln(e^x) = x$ for $x \in \mathbb{R}$
    2. $e^{\ln x} = x$ for $x \in (0, \infty)$
  - properties of logarithms: If x and y are positive numbers, and $b > 0, b \neq 1$ is real, then
    1. $\log_b(xy) = \log_b x + \log_b y$
    2. $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$
    3. $\log_b(x^r) = r \log_b x$ where r is any real number
  - properties of natural logarithms: If x and y are positive numbers, then
    1. $\ln(xy) = \ln x + \ln y$
    2. $\ln\left(\frac{x}{y}\right) = \ln x - \ln y$
    3. $\ln(x^r) = r \ln x$ where r is any real number
  - change of base
  - the basic function $f(x) = \ln x$
  - sketching and graphical transformations of logarithmic functions
- solving equations involving logarithms and exponentials
  - extraneous solutions
- Parametric equations
  - for line segment between (x_1, y_1) and (x_2, y_2): $x = (1-t)x_1 + tx_2$ $y = (1-t)y_1 + ty_2$, $0 \leq t \leq 1$
  - eliminating parameter and sketching resulting implicit function
- Solving systems of equations
  - method of substitution
  - method of elimination
- Determining the region satisfying a system of inequalities by sketching
- Conic sections
  - completing the square
  - sketching
  - derivations
  - parabolas
    * $(x-h)^2 = 4p(y-k)$ (opens up)
    * $(x-h)^2 = -4p(y-k)$ (opens down)
    * $(y-k)^2 = 4p(x-h)$ (opens to right)
    * $(y-k)^2 = -4p(x-h)$ (opens to left)
    * directrix, vertex, focus, focal length, focal width

- circles and ellipses
  * circles $(x-h)^2+(y-k)^2=r^2$
  * ellipses $\dfrac{(x-h)^2}{a^2}+\dfrac{(y-k)^2}{b^2}=1$
  * center, vertices, foci, focal axis, Pythagorean relation $c^2=a^2-b^2$ or $c^2=b^2-a^2$.
- hyperbolas
  * $\dfrac{(x-h)^2}{a^2}-\dfrac{(y-k)^2}{b^2}=1$ (opens right/left)
  * $\dfrac{(y-k)^2}{b^2}-\dfrac{(x-h)^2}{a^2}=1$ (opens up/down)
  * center, vertices, foci, focal axis, Pythagorean relation $c^2=a^2+b^2$