This review is about *process*. What I mean when I say "process" is that we have to develop confidence in the details of our solutions, since we generally do not have a solution to compare to when we are solving problems, and that is certainly true on tests. This confidence is what will serve you well during tests. And learning what you need to understand better is important in test preparation (never only redo problems you have already done as your only test preparation–you *must* be trying problems you have not done before as part of your preparation).

You can often determine if your answer is correct by

- doing the problem differently (doing a reverse decomposition or switch and solve for some f^{-1} computations)
- working backwards from the solution (reversing the transformations in a graphical transformation)
- checking using some facts (computing $f \circ f^{-1}$ and $f^{-1} \circ f$ for inverses)
- spot checking at specific values of x (this won't guarantee correct answers, but can catch errors)
- comparing with other students solutions (make sure you each solved the problem completely on your own)
- sketch using a calculator (graphical transformations)

1. Terminology (fill in with a word or phrase that completes the statement correctly)

   (a) Any set of ordered pairs is a called a ____________________.

   (b) A set of ordered pairs where no two have the same first coordinate and different second coordinates is called a ____________________.

   (c) The absolute value function is an example of a ____________________ function (think of properties of functions).

   (d) The graph of $y = f(x+4)$ is related to of the graph of $y = f(x)$ by which graphical transformation?

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   (e) If $f(-x) = -f(x)$ for all x in the domain of f, then f is an ____________________ function. (think of symmetry of functions).

   (f) The graph of $y = -f(x)$ is related to the graph of $y = f(x)$ by which graphical transformation?

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   (g) The inverse of f can be sketched from the graph of f by reflecting about the line ____________________.

   (h) If a function f is one-to-one, then f has an ____________________.

   (i) If y varies directly with the cube of x and varies inversely with the cube root of z, $y =$ ____________________.

2. Sketch by hand $f(x) = \frac{1}{4}(3 - \sqrt{-16x^2 + 16x + 5})$ and from the sketch determine the domain and range (in interval notation) of the function f.

   Hint: Interpret f as part of a circle.

3. Determine the difference quotient $\dfrac{f(x+h) - f(x)}{h}$. Simplify until substituting zero for h does not result in the indeterminant form $\dfrac{0}{0}$.

   (a) $f(x) = \dfrac{17}{4x-6}$, (b) $f(x) = \sqrt{3-2x}$, (c) $f(x) = 3x^2 - 4x$, (d) $f(x) = \dfrac{3}{\sqrt{1-x}}$.

4. Sketch the piecewise-defined function. Using your sketch, state the domain and range, and identify any intervals in which f is increasing, decreasing, or constant.

(a) $f(x) = \begin{cases} \sqrt{x-2} \text{ if } 2 \le x \le 3 \\ |-x| \text{ if } x > 3 \end{cases}$ (b) $f(x) = \begin{cases} x^3+4 \text{ if } x \le 3 \\ (-x)^{1/3} \text{ if } x > 3 \end{cases}$

5. Use transformations to graph $y = f(x)$ by transforming the graph of $y = g(x)$. Show intermediate steps in the transformation, explain each transformation in words, and finally state the domain and range of the function f.

(a) $f(x) = -3x^3 + 2,\ g(x) = x^3.$
(b) $f(x) = 2|x-3| - 17,\ g(x) = |x|.$
(c) $f(x) = -\sqrt{-x} + 2,\ g(x) = \sqrt{x}.$
(d) $f(x) = -2\sqrt[3]{x+2},\ g(x) = \sqrt[3]{x}.$

6. Given $f(x) = \frac{1}{x}$, $g(x) = \sqrt{x}$, $h(x) = x - 2$, and $w(x) = (x+2)^2$, determine the domain for the function and then simplify as much as possible.

(a) $\left(\frac{f \circ g \circ w}{w}\right)(x)$, (b) $\left(g \circ w + \frac{f}{h}\right)(x)$.

7. Is $f(x)$ even, odd, or neither? Explain how you determined your answer.

(a) $f(x) = 5x^3 - x$, (b) $f(x) = x^2 + \frac{17}{3}x$, (c) $f(x) = \sqrt{12x^2 + |x|}$.

8. Sketch $f(x) = -\frac{1}{3}\sqrt{x-2}+4$ using graphical transformations. Clearly indicate what each graphical transformation does both algebraically and in English. What is the domain and range of f? What is the x-intercept of f? What is the domain and range of f^{-1}?

9. Find the inverse function f^{-1}. Verify your answer by computing $f^{-1} \circ f$ or $f \circ f^{-1}$.

(a) $f(x) = -\sqrt[3]{3-x} + 1$, (b) $f(x) = \frac{2x+3}{7-x}$, (c) $f(x) = x^2 + 1, \quad x < 0.$

10. If J is jointly proportional to G and V, and $J = \sqrt{3}$ when $G = \sqrt{2}$ and $V = \sqrt{8}$, what is J when $G = \sqrt{6}$ and $V = 6$?

11. The time required to process a shipment of oysters varies directly with the number of pounds in the shipment and inversely with the number of workers assigned. If 3000 lbs can be processed by 6 workers in 8 hours, then how long would it take 5 workers to process 4000 lbs?

12. The volume of gas in a nitrogen shock absorber varies directly with the temperature of the gas and inversely with the amount of weight on the piston. If the volume is 10 in^3 at 80 F with a weight of 600 lbs, then what is the volume at 90 F with a weight of 800 lbs?

13. Sketch $g(x) = -x^{1/3}$ and from your sketch determine $\lim_{x\to\infty}\left[g(x)\right]$.