

The Rules of Exponentials and Logarithms

1. Choose the correct rules for logarithms and exponentials from the list below.

$\ln(A/B) = \frac{\ln(A)}{\ln B}$	T	F	$e^{a-b} = e^{a/b}$	T	F
$e^{a-b} = \frac{e^a}{e^b}$	T	F	$\ln(A) = A \ln$	T	F
$B \ln(A) = [\ln(A)]^B$	T	F	$\ln(e) = 1$	T	F
$\ln(A^B) = B \ln(A)$	T	F	$\ln(e^A + e^B) = A + B$	T	F
$e^{a+b} = e^a e^b$	T	F	$\ln(A) + \ln(B) = \ln(AB)$	T	F
$e^{a+b} = e^a + e^b$	T	F	$\ln(e^A) = A$	T	F
$\ln(A) - \ln(B) = \ln(A/B)$	T	F	$\ln(A + B) = \ln(A) + \ln(B)$	T	F
$e^{\ln A} = A$	T	F			

Discussion

Make some notes on the questions, then discuss what you wrote with a partner. Make note of anything that you and your partner disagree on, or that you had difficulty with.

- Write down in words the process to find the inverse of a function algebraically.
- Draw a sketch that shows $y = f(x) = e^x$ and $y = g(x) = \ln(x)$ on the same set of axis. Which has a horizontal asymptote? Which has a vertical asymptote? What is the domain of each function?
- Explain in words the process to change the base of a logarithm.
- Applying the power rule to $y = \log(x^2)$ yields $y = 2\log(x)$, but are these functions the same? What is the domain of each function?
- Explain the strategy for solving an equation that involves logarithms.
- Explain the strategy for solving an equation that involves exponentials.

Computation

Work out solutions, and discuss with a partner as needed. Make sure your solutions are well organized, complete, and use correct mathematical notation. Make sure any sketch you draw is labelled.

- Given $f(x) = -2\ln(-4x)$.
 - What is the domain of f ? What is the range of f^{-1} ?
 - Sketch f and f^{-1} on the same axis.
 - Find the algebraic expression for the inverse function $f^{-1}(x)$.
 - Verify your answer by showing $f(f^{-1}(x)) = x$.

8. Given $f(x) = -2\ln(-6x) + \ln(x^4)$.

- What is the domain of f ? What is the range of f^{-1} ?
- Find the algebraic expression for the inverse function $f^{-1}(x)$.
- Verify your answer by showing $f(f^{-1}(x)) = x$.

9. Graph $f(x) = -5e^x - 2$ by transforming the basic function $y = e^x$, and analyze f for:

Domain: $x \in$

Range: $y \in$

Vertical Asymptotes:

Horizontal Asymptotes:

Increasing on interval:

Decreasing on interval:

$$\lim_{x \rightarrow -\infty} (-5e^x - 2) =$$

$$\lim_{x \rightarrow \infty} (-5e^x - 2) =$$

10. A population of 200 fish is released into a lake. The population doubling time for this breed of fish is 24 months. If the plan is to allow limited fishing on the lake once the fish population exceeds 10000 fish, when should fishing be allowed to begin (give the answer exactly)?

Show all your calculations. Derive any formulas you need, do not simply plug numbers into a population growth formula you have memorized.

11. Solve the equation $R = P \frac{i}{1 - (1 + i)^{-nt}}$ for t .

12. Solve the equation $\frac{2^x - 2^{-x}}{3} = 4$ algebraically for x .

13. A turkey with a temperature of 40F is moved to an oven at 350F. After 4 hours, the internal temperature of the turkey is 170F. If the turkey is done when its temperature is 185F, how much longer must it cook? Use Newton's Law of Cooling

$$D = D_0 e^{kt},$$

where D_0 is the initial difference in temperature and D is difference at time t . The parameter k is a constant that depends on the object and its surroundings.

14. Solve the equation $\ln(x) - \ln(x + 1) = \ln(x + 3) - \ln(x + 5)$ algebraically for x .

15. Solve the equation $e^{3\ln(x^2) - 2\ln(x)} = \ln(e^{16})$ algebraically for x .

16. Solve the equation $\log_{10}(x + 1) - \log_{10}(x) = 3$ algebraically for x .