Note: You can expect other types of questions on the test than the ones presented here! Make sure you also review the practice problems from each section and webwork problems.

Questions

Example 1 The pendulum on a grandfather clock travels through an angle of 24 degrees (from one side to the other). If the chain is 4.5 feet long, what distance does the weight at the end of the pendulum travel in one oscillation (returning to its starting point)?

Example 2 Find $\sec\theta$ if $\tan\theta = \dfrac{1}{5}$ and $\sin\theta < 0$.

Example 3 Explain why $\cos\pi/4 = \frac{1}{\sqrt{2}}$.

Example 4 What is $\csc\pi/3$?

Example 5 Find exactly the cosecant of an angle in standard position that has a terminal side that ends at the point $P(-12, 12)$.

Example 6 Explain why $\cot(-3\pi/4) = 1$. To get full marks, you must have a well explained process that includes a coordinate system that shows the angle $-3\pi/4$, the appropriate special triangle that helps you label x y and r in the coordinate system, and the definition of cotangent.

Solutions

Example 1 The distance the weight travels is going to be along the arc of a circle, so we need the arc length formula to figure this out. The arc length formula is $s = r\theta$, where s is the arc length, r is the radius of the circle, and θ is the angle measured in radians.

$$\frac{24 \text{ degrees}}{180 \text{ degrees}} = \frac{x \text{ radians}}{\pi \text{ radians}} \Rightarrow x = \frac{24\pi}{180} \text{ radians} = \frac{2\pi}{15} \text{ radians} .$$

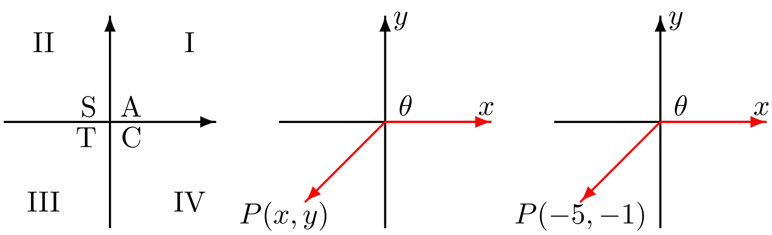
The length the weight travels from one side to other is

$$\begin{aligned} s &= r\theta \\ &= 4.5 \text{ feet } \times \frac{2\pi}{15} \text{ radians } = \frac{9\pi}{15} \text{ feet } = \frac{3\pi}{5} \text{ feet} \end{aligned}$$

This is half an oscillation, so the pendulum travels $\frac{6\pi}{5}$ ft in one oscillation.

Example 2 Find $\sec\theta$ if $\tan\theta = \dfrac{1}{5}$ and $\sin\theta < 0$.

Since $\sin\theta$ is less than zero, we must be in either Quadrant III or IV.
Since $\tan\theta$ is greater than zero we must be in either Quadrant I or III.
Therefore, the angle θ has a terminal side in Quadrant III.



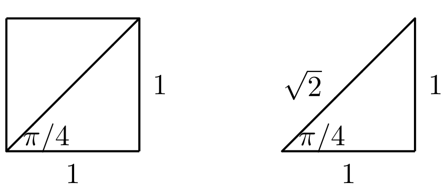
Since $\tan\theta = \dfrac{y}{x} = \dfrac{1}{5} = \dfrac{-1}{-5}$, we have $x = -5$, $y = -1$.

The distance $r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (-5)^2} = \sqrt{26}$.

$$\text{Therefore, } \sec\theta = \frac{1}{\cos\theta} = \frac{r}{x} = \frac{\sqrt{26}}{-5} = -\frac{\sqrt{26}}{5}.$$

Example 3 Explain why $\cos \pi/4 = \frac{1}{\sqrt{2}}$

Consider the square given below.



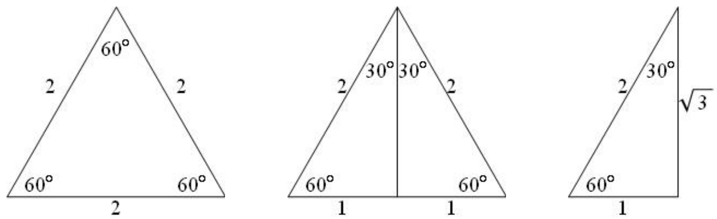
The angle here must be $\pi/4$ radians, since this triangle is half of a square of side length 1.

Now, we can write down all the trig functions for an angle of $\pi/4$ radians = 45 degrees:

$$\cos\left(\frac{\pi}{4}\right) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}}$$

Example 4 What is $\csc \pi/3$?

This angle, $\pi/3$, is one of our special angles.

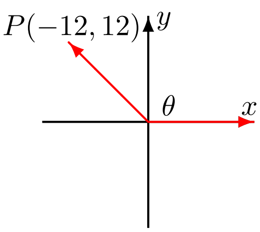


Recall that $\pi/3 = 60°$.

$\sin \pi/3 = \text{opp}/\text{hyp} = \sqrt{3}/2$.

$\csc \pi/3 = 1/\sin \pi/3 = 2/\sqrt{3}$.

Example 5 Find exactly the cosecant of an angle in standard position that has a terminal side that ends at the point $P(-12, 12)$.



So $x = -12$, $y = 12$, and $r = 12\sqrt{2}$ ($r = \sqrt{(-12)^2 + (12)^2}$).

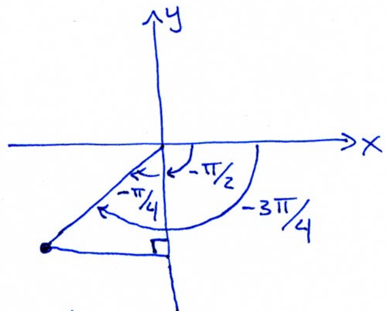
$\sin \theta = \dfrac{y}{r}$. $\qquad \csc \theta = \dfrac{1}{\sin \theta} = \dfrac{r}{y} = \dfrac{12\sqrt{2}}{12} = \sqrt{2}$.

Example 6 Explain why $\cot(-3\pi/4) = 1$. To get full marks, you must have a well explained process that includes a coordinate system that shows the angle $-3\pi/4$, the appropriate special triangle that helps you label x y and r in the coordinate system, and the definition of cotangent.

sketch:

$-\frac{3\pi}{4}$ is < 0, so we measure the angle clockwise.

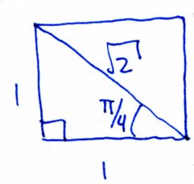
$$-\frac{3\pi}{4} = -\frac{\pi}{2} - \frac{\pi}{4}$$

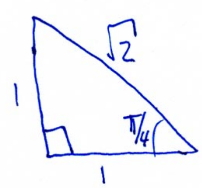


We are in Quadrant III.

pull out the triangle:

$\pi/4$ is one of our special angles:





Comparing these two triangles, we have

$x = 1$
$y = 1$
$r = \sqrt{2}$

But, in Quadrant III, $x < 0$ and $y < 0$.

so we have $x = -1$
$y = -1$
$r = \sqrt{2}$.

From the definitions, $\cot(-3\pi/4) = \dfrac{1}{\tan(-3\pi/4)}$

$$= \frac{1}{(y/x)}$$

$$= \frac{x}{y} = \frac{-1}{-1} = 1.$$