Questions

1. Match the function on the left with the sketch on the right (no work needs to be shown here, fill in directly on this sheet).

$y = -\sec x$ matches graph D

$= -\frac{1}{\cos x}$

$y = -\tan x$ matches graph H

Note minus sign!

$y = \sin x^2$ matches graph A

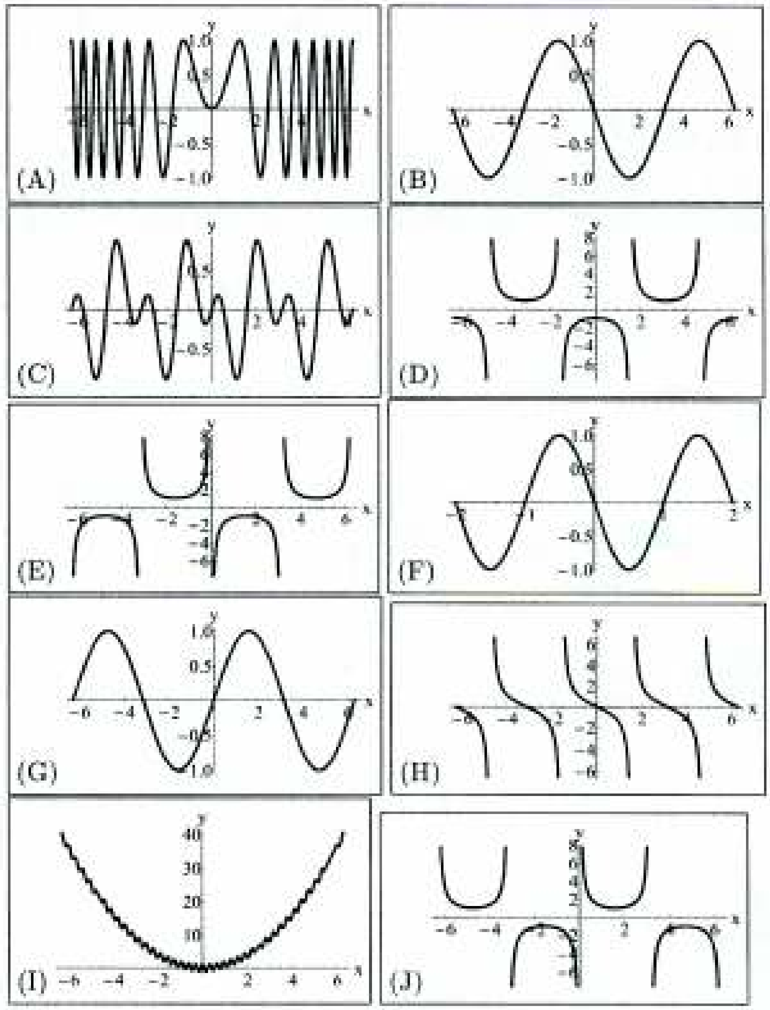
Range $[-1, 1]$, but not a sinusoid.

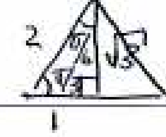
$y = \cos(3x) + \sin x$ matches graph C

periodic, but not a sinusoid.

$y = x^2 + \cos(20x)$ matches graph I

oscillate around $y = x^2$.





2. Answer True or False (no work needs to be shown here, fill in directly on this sheet):

Note: explanation is not required on the test

$\sin(\pi/4) = \frac{1}{\sqrt{2}}$ .... T F

$\cos(\pi/3) = \frac{\sqrt{3}}{2}$ .... $\cos \pi/3 = 1/2$ .... T F

$\tan(\pi/6) = \frac{1}{2}$ .... $\tan \pi/6 = 1/\sqrt{3}$ .... T F

Sine is an even function .... Whoops! ~~T~~ F

$y = \frac{1}{x^2+1}\cos x$ exhibits damping as $x \to \infty$ .... As $x \to \infty$ $\frac{1}{x^2+1} \to 0$. .... T F

$\cos t \to \infty$ as $t \to \infty$ .... cos t oscillates between -1 and +1. .... T F

$\sin t = \frac{1}{\cos t}$ .... $\sin t = \frac{1}{\csc t}$ .... T F

$\tan t$ has period $\pi/2$ .... tan t has period π .... T F

$\tan t = \frac{\sin t}{\cos t}$ .... T F

$\csc t$ has a vertical asymptote at $t = \pi$ .... T F

$y = \cos(2x) + \sin(4x)$ is periodic .... Yes, but it isn't a sinusoid! .... T F

$y = \cos(2x) + \sin(4x)$ is a sinusoid .... (cos2x and sin4x have different periods) .... T F

$y = 2\cos(4x) + \sin(4x)$ is a sinusoid .... T F

The domain of $y = \sin\theta$ is $\theta \in [-1, 1]$ .... domain is all real numbers .... T F

The range of $y = -7\sin\theta$ is $y \in [-7, 7]$ .... T F

3. Sketch the sinusoid $y = -5\sin(2x + 7) - 6$. Determine the Amplitude, Period, and Phase Shift of the sinusoid. ~~Clearly label the location of a local minimum on your sketch.~~

4. Sketch the sinusoid $y = 3\sin(\pi(2x + 1))$. Determine the Amplitude, Period, and Phase Shift of the sinusoid. Clearly label the location of a local minimum on your sketch.

5. Solve $\tan t = -\sqrt{3}$ for all values of t.

6. Solve $\sec t = \frac{2}{\sqrt{3}}$ for all values of t.

3) $y = -5\sin(2x+7) - 6$ ← vertical shift down 6 units.

Amplitude $= |-5| = 5$

$0 \le 2x+7 \le 2\pi$

$-7 \le 2x \le -7+2\pi$

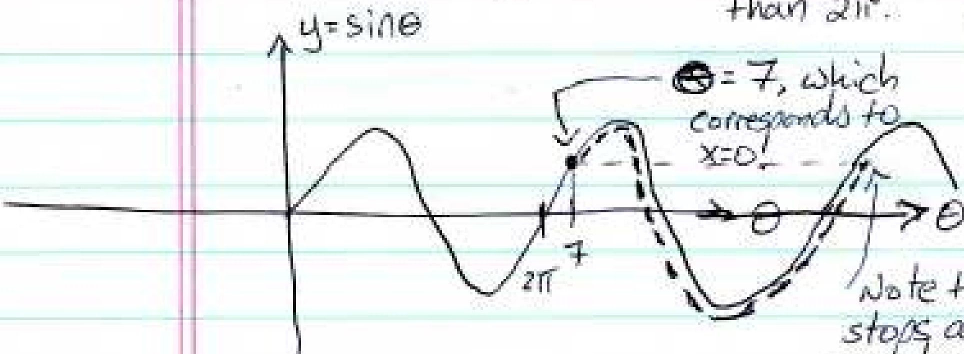
$\underbrace{-\tfrac{7}{2} \le x \le -\tfrac{7}{2}}_{\text{phase shift} = -7/2} + \pi$ ← period π

When $x=0$, $y = -5\sin(2\cancel{x}^{0}+7) - 6$

$= -5\sin(7) - 6$

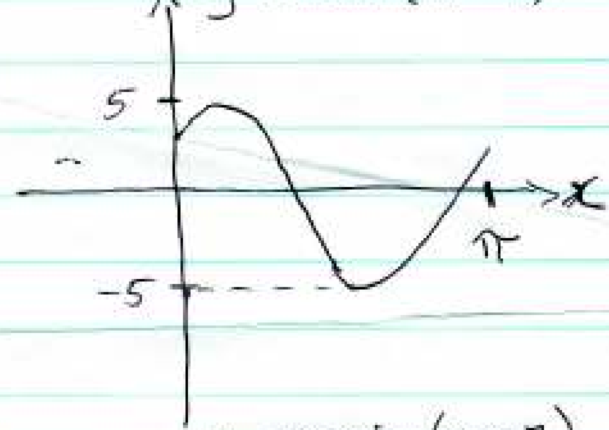
Note: minus sign. $\Rightarrow$ flip across x-axis!

Note: 7 is just a bit bigger than 2π.

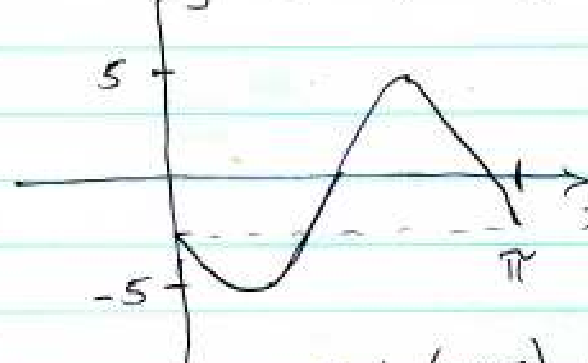


There is a lot going on, so I will do a few sketches

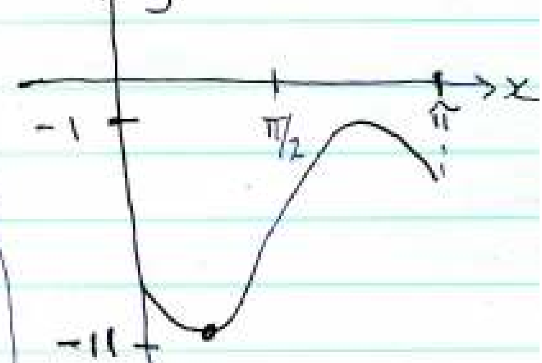
$y = 5\sin(2x+7)$



$y = -5\sin(2x+7)$



$y = -5\sin(2x+7) - 6$



4) $y = 3\sin(\pi(2x+1))$

Amplitude $= 3$

$0 \le \pi(2x+1) \le 2\pi$

$0 \le 2x+1 \le 2$

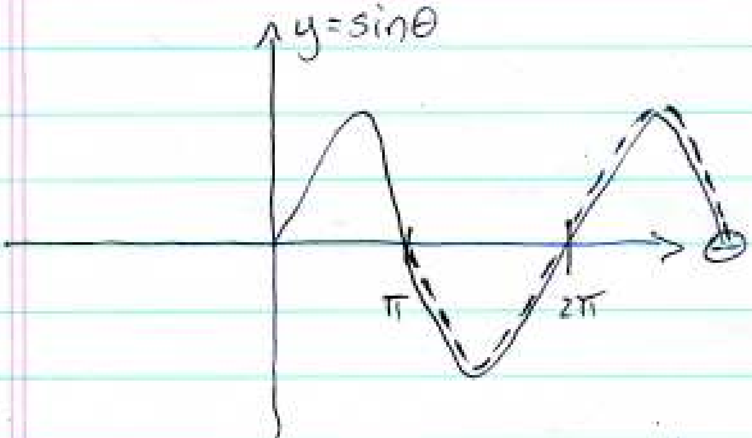
(don't simplify!) $-1 \le 2x \le -1+2$

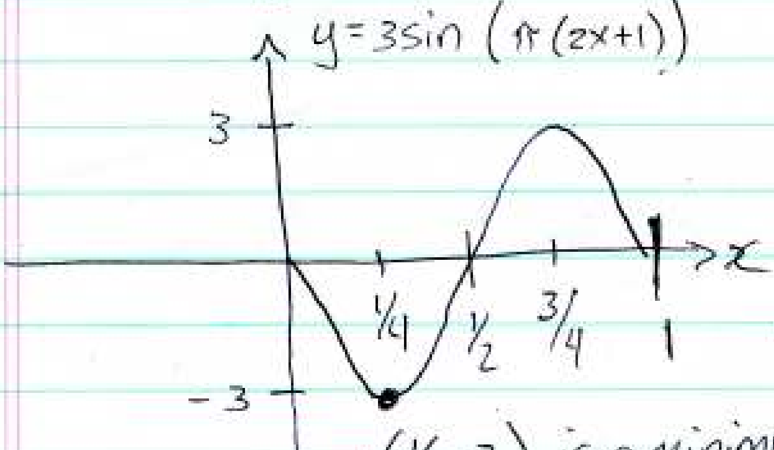
$-\frac{1}{2} \le x \le -\frac{1}{2} + 1$ ← period $= 1$

phase shift $= -1/2$

when $x=0$, $y = 3\sin(\pi(2\overset{0}{\cancel{x}}+1))$

$= 3\sin(\pi)$



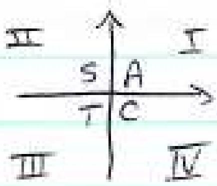


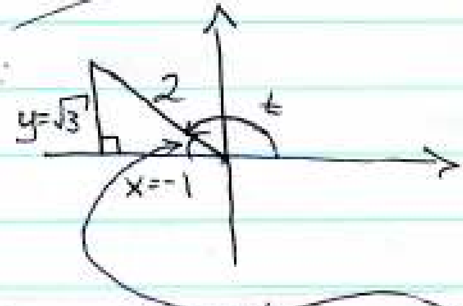
$(1/4, -3)$ is a minimum.

5) $\tan t = -\sqrt{3} = y/x \Rightarrow$ which Quadrant? If in QII, $y = \sqrt{3}$, $x = -1$

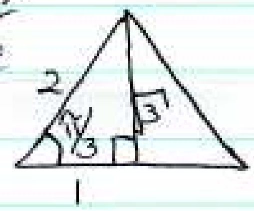
Locate
Label
Answer

$\tan t < 0$ in II or IV. Pick QII:





Now Compare to

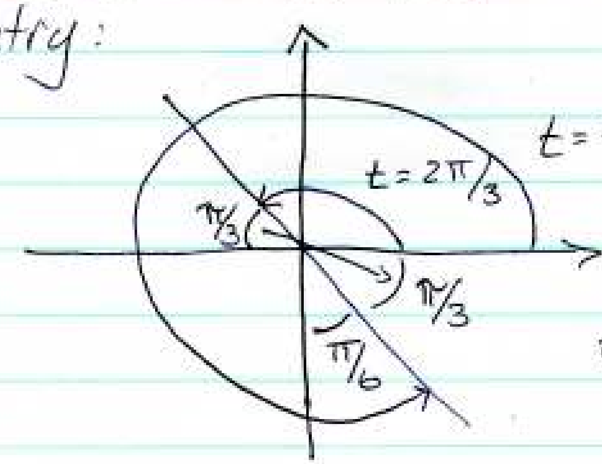


This tells us this angle is $\pi/3$.

OK, Now, $t = \pi - \pi/3 = 2\pi/3$.

Tangent has period π, so $t = 2\pi/3 \pm n\pi$, $n = 0, 1, 2, 3, \ldots$ are solutions.

There are also solutions in QIV, which we can find by geometry:



$t = 3\pi/2 + \pi/6 = \dfrac{10\pi}{6} = \dfrac{5\pi}{3}$.

OR

$t = \dfrac{2\pi}{3} + \pi = \dfrac{5\pi}{3}$.

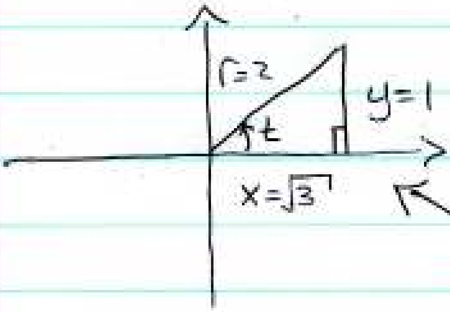
other solutions are $t = \dfrac{5\pi}{3} \pm n\pi$, $n = 0, 1, 2, 3, \ldots$

Note: tangent is special, since this second set of solutions are contained in $t = \frac{2\pi}{3} + n\pi$, $n = 1$!
This was since tangent has period π; this doesn't happen for sine and cosine. If you sketched $\tan t$ you would see this.

6) $\sec t = \frac{2}{\sqrt{3}} \Rightarrow \cos t = \frac{\sqrt{3}}{2}$.

Locate
Label
Answer

$\cos t > 0$ in QI or QIV. Pick QI to start with.

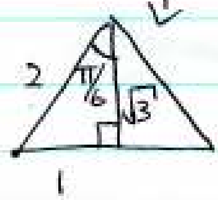


$\cos t = \frac{\sqrt{3}}{2} = \frac{x}{r} \Rightarrow x = \sqrt{3}$

$r = 2$

$y = \sqrt{r^2 - x^2} = 1$

Compare with

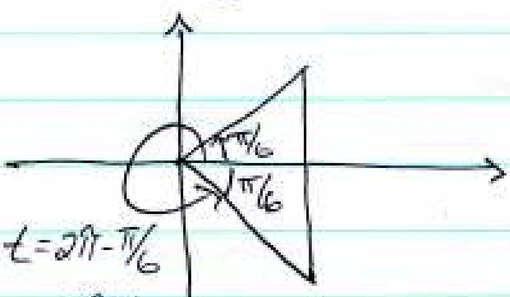


$t = \pi/6$! Cosine has period 2π.

solutions

$t = \frac{\pi}{6} \pm 2n\pi$, $n = 0, 1, 2, \ldots$

Use Geometry to find solutions in QIV:



$t = 2\pi - \pi/6$
$= 11\pi/6$.

Note I would accept

$t = \frac{11\pi}{6} \pm 2n\pi$ $n = 0, 1, 2, \ldots$

OR

$t = -\pi/6 \pm 2n\pi$ $n = 0, 1, 2, \ldots$

since these both give same terminal side.