Note: You can expect other types of questions on the test than the ones presented here!

Questions

Example 1. Find the equation of the line passing through the point $(0, 0)$ that is perpendicular to the line passing through the points $(0, 1)$ and $(1, 2)$.

Example 2. Solve the equation $3x^2 - 4x = 1/2$ by completing the square.

Example 3. Determine the domain of the function $f(x) = \dfrac{4}{x\sqrt{2x-8}}$

Example 4. Given $w(t) = \dfrac{7 - 2t^{5/2}}{t\sqrt{1+4t^4}}$, what function does the function $w(t)$ approach for large $t > 0$? Simplify as much as possible.

Example 5. Given $f(t) = \dfrac{4t^2 + 3t - 1}{(t+1)(t-1)}$, what is the end behaviour of $f(t)$ for large $t > 0$? If $f(t)$ has a horizontal asymptote, what is the horizontal asymptote? If $f(t)$ has vertical asymptotes, what are the vertical asymptotes?

Example 6. Given $f(x) = \sqrt{2x-3}$, simplify the quantity $\dfrac{f(x+h) - f(x)}{h}$ so that substitution of $h = 0$ does not give $\frac{0}{0}$.

Example 7. Given $f(x) = \dfrac{1}{2+x}$, simplify the quantity $\dfrac{f(x+h) - f(x-h)}{2h}$ so that substitution of $h = 0$ does not give $\frac{0}{0}$.

Example 8. Determine whether the following function is even, odd, or neither. Use the algebraic technique to determine if a function is even or odd, rather than attempting to sketch the function.

$$g(x) = \frac{4x^3 - x}{2x^3 - x}$$

Example 9. Find a formula $f^{-1}(x)$ for the inverse of the function (you do not have to discuss domain and range):

$$f(x) = \frac{1 + 7x}{4 - x}$$

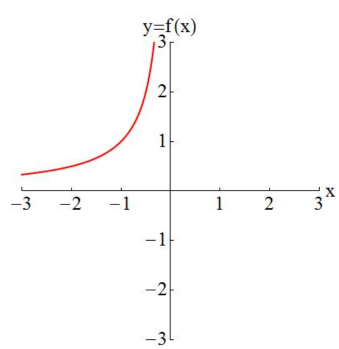
Example 10. Find $f^{-1}(x)$ given $f(x) = 3 + \frac{2}{x}$, $x > 0$. What is the domain of $f^{-1}(x)$?

Example 11. Sketch the graph of the piecewise defined function f, and label three (x, y) ordered pairs on the graph. From your graph, what is the range of f?

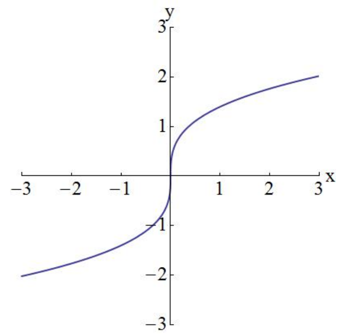
$$f(x) = \begin{cases} x^2 - 2 & \text{if } x > 0 \\ -1 - x & \text{if } x < 0 \\ 120 & \text{if } x = 0 \end{cases}$$

Example 12. Given the functions $f(x) = x - 3$ and $g(x) = x^2$, determine the composition $(g \circ f \circ f)(x)$ (simplify as much as possible). You do not have to discuss domains.

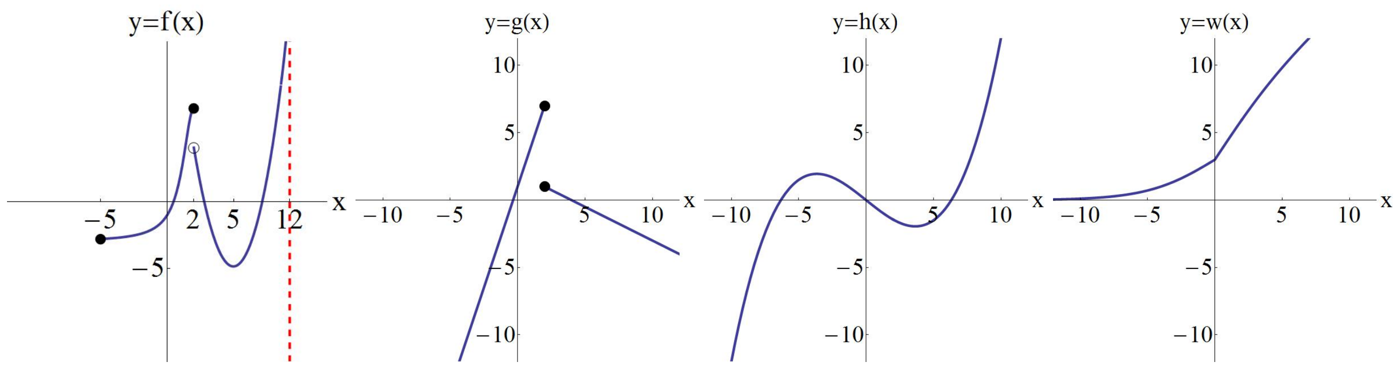
Example 13. Given below is a sketch of the function $y = f(x)$. Add to this sketch a sketch of $y = -f(-x)$.



Example 14. Given below is a sketch of the function $y = f(x)$. Add to this sketch a sketch of $y = f^{-1}(x)$.



Example 15. Answer questions (i)–(x) based on the following graphs.



(i) $f(x)$ is continuous for $x \in [2, 12)$ .... T F

(ii) $f(x)$ has a vertical asymptote given by $x = 12$ .... T F

(iii) $f(x)$ is a function with domain $x \in [-5, 12)$ .... T F

(iv) $f(x)$ is bounded above .... T F

(v) $g(x)$ is not a function .... T F

(vi) $h(-x) = -h(x)$ .... T F

(vii) $h(x)$ is an odd function .... T F

(viii) $h(x)$ is a one-to-one function .... T F

(ix) $w(x)$ is a bounded function .... T F

(x) $w(x)$ has two horizontal asymptotes .... T F

Solutions

Ex 1 Slope of line through $(0,1)$ and $(1,2)$ is

$$m = \frac{\Delta y}{\Delta x} = \frac{2-1}{1-0} = 1$$

Slope of line perpendicular will be negative reciprocal $\Rightarrow$ $\boxed{m = -\frac{1}{1} = -1}$.

Equation of line with slope $m=-1$ passing through $(0,0)$ is $y = mx + b$

$y = -x + b$ use $(0,0)$ to determine b

$0 = 0 + b \Rightarrow b = 0.$

$$\boxed{y = -x}$$

Ex 2

$$3x^2 - 4x = \frac{1}{2}$$

$$x^2 - \frac{4}{3}x = \frac{1}{6}$$

$$\underbrace{x^2 - \frac{4}{3}x + \left(\frac{4}{6}\right)^2} - \left(\frac{4}{6}\right)^2 = \frac{1}{6}$$

$$\left(x - \frac{4}{6}\right)^2 = \frac{1}{6} + \left(\frac{4}{6}\right)^2$$

$$= \frac{1}{6} + \frac{16}{36}$$

$$= \frac{6}{36} + \frac{16}{36} = \frac{22}{36} = \frac{11}{18}$$

$$x - \frac{4}{6} = \pm\sqrt{11/18}$$

$$x = \frac{4}{6} \pm \sqrt{\frac{11}{18}}$$

Ex 3 Division by zero means $x \neq 0$ or $2x - 8 \neq 0$

$2x \neq 8$

$x \neq 4$.

Square Root Requires $2x - 8 \geq 0$ to get a real number out.

$2x \geq 8$

$x \geq 4$.

Put this info on number line:

Domain $x > 4$ or $x \in (4, \infty)$.

$\boxed{\text{Ex 4}}$ For large $t>0$, $7-2t^{5/2} \sim -2t^{5/2}$ and $t\sqrt{1+4t^4} \sim t\sqrt{4t^4}$
$= t(2t^2)$
$= 2t^3$.

so $w(t) \sim \dfrac{-2t^{5/2}}{2t^3}$ when $t>0$ is large

$= -t^{5/2-3}$

$= -t^{-1/2}$ or $w(t) \sim -\dfrac{1}{\sqrt{t}}$ when $t>0$ is large.

$\boxed{\text{Ex 5}}$ When t is large, $f(t) \sim \dfrac{4t^2}{(t)(t)} = 4$ (take dominant term in each polynomial).

This means $\lim_{t\to\infty} f(t) = 4$, so f has a horizontal asymptote of $y=4$.

There are two vertical asymptotes, $x=\pm 1$ (where we get division by zero).

$\boxed{\text{Ex 6}}$ $f(x+h) = \sqrt{2(x+h)-3} = \sqrt{2x+2h-3}$

$\dfrac{f(x+h)-f(x)}{h} = \dfrac{\sqrt{2x+2h-3}-\sqrt{2x-3}}{h}$ substituting $h=0$ would give $\frac{0}{0}$, so we have to rationalize the numerator.

$= \dfrac{\left(\sqrt{2x+2h-3}-\sqrt{2x-3}\right)}{h} \cdot \dfrac{\left(\sqrt{2x+2h-3}+\sqrt{2x-3}\right)}{\left(\sqrt{2x+2h-3}+\sqrt{2x-3}\right)}$

$= \dfrac{(\cancel{2x}+2h-\cancel{3})-(\cancel{2x}-\cancel{3})}{h\left(\sqrt{2x+2h-3}+\sqrt{2x-3}\right)}$

$= \dfrac{2\cancel{h}}{\cancel{h}\left(\sqrt{2x+2h-3}+\sqrt{2x-3}\right)}$

$= \dfrac{2}{\sqrt{2x+2h-3}+\sqrt{2x-3}}$

$\boxed{\text{Ex 7}}$ $f(x+h) = \dfrac{1}{2+x+h}$ $f(x-h) = \dfrac{1}{2+x-h}$

$\dfrac{f(x+h)-f(x-h)}{2h} = \dfrac{1}{2h}\left[f(x+h)-f(x-h)\right]$

$= \dfrac{1}{2h}\left[\dfrac{1}{2+x+h} - \dfrac{1}{2+x-h}\right]$ get a common denominator.

$= \dfrac{1}{2h}\left[\dfrac{\cancel{2}+\cancel{x}-h-\cancel{2}-\cancel{x}-h}{(2+x+h)(2+x-h)}\right]$

$= \dfrac{-\cancel{2h}}{\cancel{2h}(2+x+h)(2+x-h)}$

$= \dfrac{-1}{(2+x+h)(2+x-h)}$

Ex 8)

$$g(-x) = \frac{4(-x)^3 - (-x)}{2(-x)^3 - (-x)} = \frac{-4x^3 + x}{-2x^3 + x} = \frac{\cancel{-1}(4x^3 - x)}{\cancel{-1}(2x^3 - x)} = \frac{4x^3 - x}{2x^3 - x} = g(x)$$

since $g(-x) = g(x)$, g is even.

Ex 9) $y = \dfrac{1+7x}{4-x}$

interchange x and y: $x = \dfrac{1+7y}{4-y}$

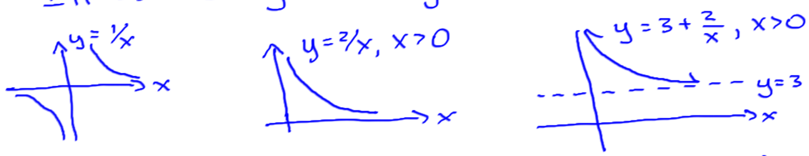
solve for y:

$$x(4-y) = 1+7y$$
$$4x - yx = 1 + 7y$$
$$-yx - 7y = 1 - 4x$$
$$y(-x-7) = 1 - 4x$$

$\rightarrow$ $y = \dfrac{1-4x}{-x-7}$

$$f^{-1}(x) = \frac{1-4x}{-x-7} = \frac{4x-1}{x+7}.$$

Ex 10) Deal with Domain first.
I'll do this by sketching $f(x)$.



Get f^{-1}: $y = 3 + \dfrac{2}{x}$

interchange x and y: $x = 3 + \dfrac{2}{y}$

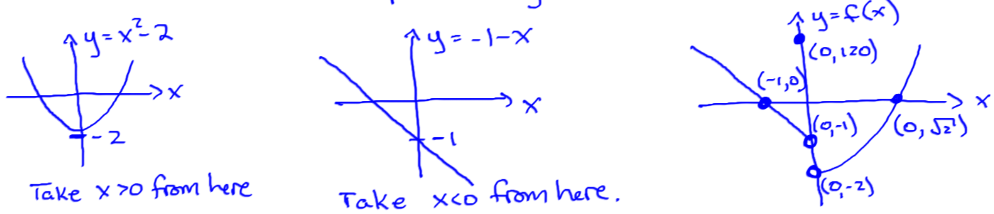
solve for y: $x - 3 = \dfrac{2}{y}$

$$y = f^{-1}(x) = \frac{2}{x-3}, \quad x \in (3, \infty).$$

f: Domain $(0, \infty)$
Range $(3, \infty)$ (from sketch)

so f^{-1}: Domain $(3, \infty)$
Range $(0, \infty)$.

Ex 11) Sketch each piece, then put it together at the end.



Take $x > 0$ from here

Take $x < 0$ from here.

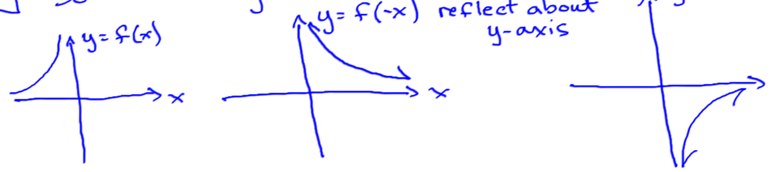
(Not to scale)

Range of f is (from the sketch)
$y \in (-2, \infty)$

Ex 12

$$
\begin{aligned}
(g \circ f \circ f)(x) &= g(f(f(x))) \\
&= g(f(x-3)) \\
&= g(x-3-3) \\
&= g(x-6) \\
&= (x-6)^2 \\
&= x^2-12x+36
\end{aligned}
$$

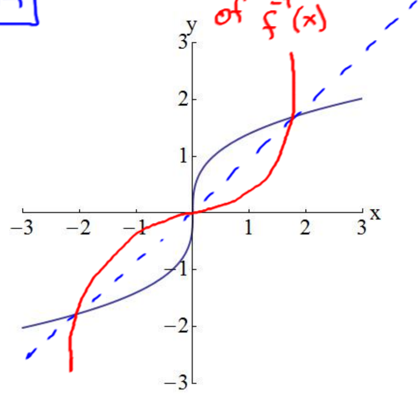
Ex 13 Do this in stages:



Ex 14

rough sketch of $f^{-1}(x)$

$y = x$ (reflect about this line)



Ex 15

i) F (not continuous at $x = 2$)

ii) T

iii) T

iv) False (not bounded above as $x \to 12$ from left)

v) T (fails vertical line test)

vi) T (h is odd)

vii) T (h is odd)

viii) F (fails horizontal line test)

ix) F (appears to be unbounded on the right as $x \to \infty$)

x) F (only one horizontal asymptote).