Note: You can expect other types of questions on the test than the ones presented here!

Questions

Example 1 State the amplitude, period, and frequency of the sinusoid, and the phase shift and vertical translation.

$$y = \frac{2}{3}\sin\left(\frac{6x-3}{4}\right) + 1.$$

Example 2 Find $\sec\theta$ if $\tan\theta = \dfrac{1}{5}$ and $\sin\theta < 0$.

Example 3 Explain why $\cos\pi/4 = \frac{1}{\sqrt{2}}$

Example 4 What is $\csc\pi/3$?

Example 5 Find exactly the cosecant of an angle in standard position that has a terminal side that ends at the point $P(-12, 12)$.

Example 6 What is $\lim_{x\to 0} \arcsin x$?

Example 7 Find the exact value of $\arccos(-1/2)$.

Example 8 Find an algebraic expression equivalent to the expression $\tan(\arccos(z+1))$.

Example 9 Explain why $\cot(-3\pi/4) = 1$.

Solutions

Example 1 State the amplitude, period, and frequency of the sinusoid, and the phase shift and vertical translation.

$$y = \frac{2}{3}\sin\left(\frac{6x-3}{4}\right) + 1.$$

The amplitude of the sinusoid is $\left|\frac{2}{3}\right| = \frac{2}{3}$.

There is a vertical translation of $+1$ units upwards.

The sine function has period 2π. Therefore, $\sin\theta$ completes one period if $0 \le \theta \le 2\pi$. Therefore, the sinusoid will complete one period if

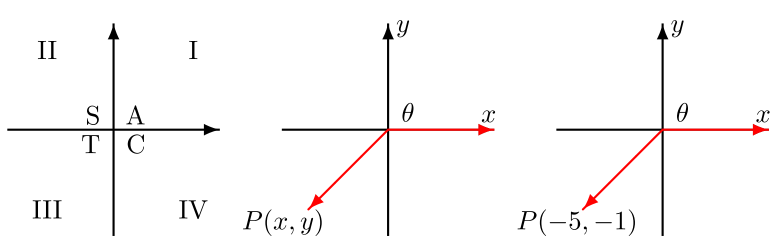
$$\begin{gathered}
0 \le \frac{6x-3}{4} \le 2\pi \\
0 \le 6x - 3 \le 8\pi \\
3 \le 6x \le 3 + 8\pi \\
\frac{1}{2} \le x \le \frac{1}{2} + \frac{4\pi}{3}
\end{gathered}$$

This function has period $\dfrac{4\pi}{3}$, and a phase shift of $1/2$ units.

The frequency is $\dfrac{3}{4\pi}$.

Example 2 Find $\sec\theta$ if $\tan\theta = \dfrac{1}{5}$ and $\sin\theta < 0$.

Since $\sin\theta$ is less than zero, we must be in either Quadrant III or IV.
Since $\tan\theta$ is greater than zero we must be in either Quadrant I or III.
Therefore, the angle θ has a terminal side in Quadrant III.



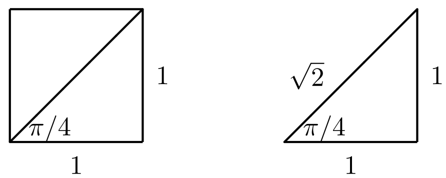
Since $\tan\theta = \dfrac{y}{x} = \dfrac{1}{5} = \dfrac{-1}{-5}$, we have $x = -5$, $y = -1$.

The distance $r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (-5)^2} = \sqrt{26}$.

$$\text{Therefore, } \sec\theta = \frac{1}{\cos\theta} = \frac{r}{x} = \frac{\sqrt{26}}{-5} = -\frac{\sqrt{26}}{5}.$$

Example 3 Explain why $\cos \pi/4 = \frac{1}{\sqrt{2}}$

Consider the square given below.



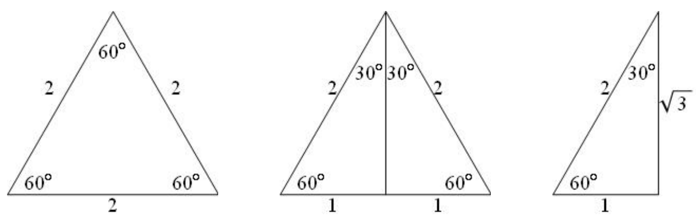
The angle here must be $\pi/4$ radians, since this triangle is half of a square of side length 1.

Now, we can write down all the trig functions for an angle of $\pi/4$ radians = 45 degrees:

$$\cos\left(\frac{\pi}{4}\right) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}}$$

Example 4 What is $\csc \pi/3$?

This angle, $\pi/3$, is one of our special angles.

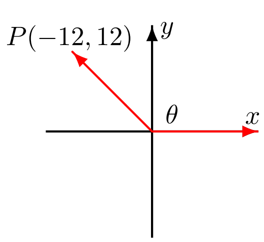


Recall that $\pi/3 = 60°$.

$\sin \pi/3 = \text{opp}/\text{hyp} = \sqrt{3}/2$.

$\csc \pi/3 = 1/\sin \pi/3 = 2/\sqrt{3}$.

Example 5 Find exactly the cosecant of an angle in standard position that has a terminal side that ends at the point $P(-12, 12)$.



So $x = -12$, $y = 12$, and $r = 12\sqrt{2}$ ($r = \sqrt{(-12)^2 + (12)^2}$).

$\sin\theta = \dfrac{y}{r}$. $\qquad \csc\theta = \dfrac{1}{\sin\theta} = \dfrac{r}{y} = \dfrac{12\sqrt{2}}{12} = \sqrt{2}$.

Example 6 What is $\lim_{x \to 0} \arcsin x$?

We could sketch the arcsine function, and read the limit off of the sketch. Or we could do the following.

Let $\theta = \arcsin x$.
Then $\sin \theta = x$.

If we take the limit as x approaches zero, that means we want to solve the equation $\sin \theta = 0$.

The angle which has a sine of zero is $\theta = 0$.

Note we cannot use $\theta = \pi, 2\pi$, etc. since these values of theta are not in the restricted domain of the sine function which we must use when we construct the inverse sine function.

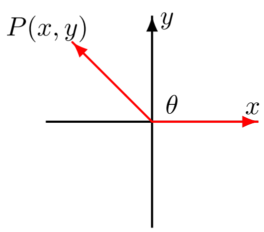
Therefore, $\lim_{x \to 0} \arcsin x = \lim_{x \to 0} \theta = 0$.

Example 7 Find the exact value of $\arccos(-1/2)$.

We could sketch the arcsine function, and attempt to read the value off of the sketch. Or we could do the following.

Let $\theta = \arccos(-1/2)$.
Then $\cos \theta = -1/2$.

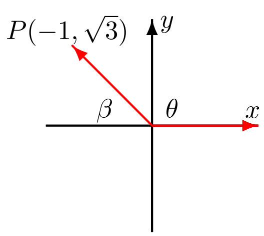
Since the cosine is less than zero, θ must be in Quadrant II. The angle θ can't be in Quadrant III since the arcosine results in an angle only in Quadrant I or II (range of arccosine function is $[0, \pi]$).



$\cos \theta = -\frac{1}{2} = \frac{-1}{2} = \frac{x}{r}$, so $x = -1$, $r = 2$.

Using the Pythagorean theorem, we have $y = \sqrt{r^2 - x^2} = \sqrt{(2)^2 - (-1)^2} = \sqrt{3}$.

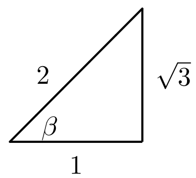
Since we are told to find the value exactly, we expect this problem to have one of our two special triangles in it somewhere.



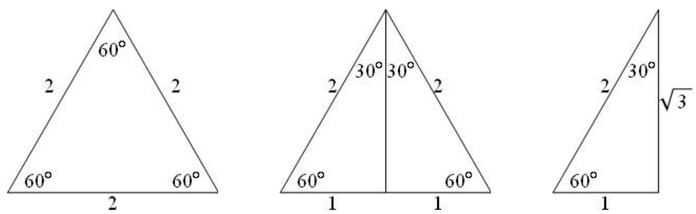
If we can figure out β, we can find θ by using $\theta + \beta = \pi$.

Construct a reference triangle for the angle β (ignoring signs at this point, since we only need to know what β is):

$\cos\beta = \frac{1}{2} = \frac{\text{adj}}{\text{hyp}}$.



Compare with the following:



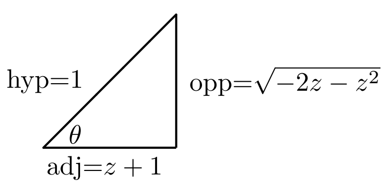
So $\beta = 60^\circ = \pi/3$.

Therefore, $\theta = \pi - \pi/3 = 2\pi/3$.

Example 8 Find an algebraic expression equivalent to the expression $\tan(\arccos(z+1))$.

To simplify this let $\theta = \arccos(z+1)$. This means $\cos\theta = z + 1 = \frac{z+1}{1} = \frac{\text{adj}}{\text{hyp}}$.

Construct a reference triangle



The length of the opposite side was found using the Pythagorean theorem:

$$\text{opp} = \sqrt{1^2 - (z+1)^2} = \sqrt{1 - (z^2 + 2z + 1)} = \sqrt{-z^2 - 2z}.$$

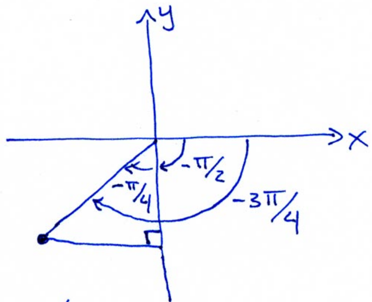
Therefore, we have

$$\tan\big(\arccos(z+1)\big) = \tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{-z^2 - 2z}}{z+1}.$$

Example 9 Explain why $\cot(-3\pi/4) = 1$.

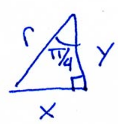
sketch: $-\frac{3\pi}{4}$ is < 0, so we measure the angle clockwise.

$$-\frac{3\pi}{4} = -\frac{\pi}{2} - \frac{\pi}{4}$$

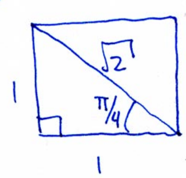


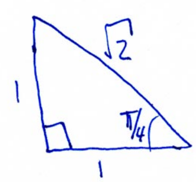
We are in Quadrant III.

pull out the triangle:



$\pi/4$ is one of our special angles:





Comparing these two triangles, we have

$x = 1$
$y = 1$
$r = \sqrt{2}$

But, in Quadrant III, $x < 0$ and $y < 0$.

so we have $x = -1$
$y = -1$
$r = \sqrt{2}$.

From the definitions, $\cot(-3\pi/4) = \frac{1}{\tan(-3\pi/4)}$

$$= \frac{1}{(y/x)}$$

$$= \frac{x}{y} = \frac{-1}{-1} = 1.$$