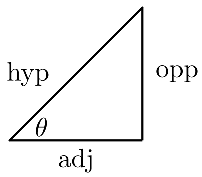
1011 Precalculus Chapter 4: Concepts to Review

- angular measure, degree, radians, arc length formula

  Basic relation: 180 degrees $= \pi$ radians .
  Arc Length: $s = r\theta$.

- right triangle trigonometry, Pythagorean theorem (acute angles)



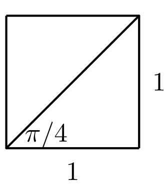
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$$
$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} \quad \cot\theta = \frac{\text{adjacent}}{\text{opposite}}$$

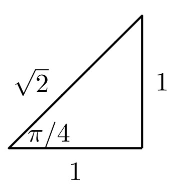
- special triangles (4.2)

  The six basic trigonometric functions relate the angle θ to ratios of the length of the sides of the right triangle. For certain triangles, the trig functions of the angles can be found geometrically. These special triangles occur frequently enough that it is expected that you learn the value of the trig functions for the special angles.

  A 45-45-90 Triangle

  Consider the square given below.





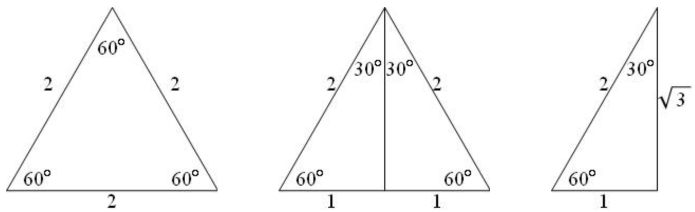
The angle here must be $\pi/4$ radians, since this triangle is half of a square of side length 1.

Now, we can write down all the trig functions for an angle of $\pi/4$ radians = 45 degrees:

$$\sin\left(\frac{\pi}{4}\right) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} \qquad \csc\left(\frac{\pi}{4}\right) = \frac{1}{\sin\left(\frac{\pi}{4}\right)} = \sqrt{2}$$
$$\cos\left(\frac{\pi}{4}\right) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{\sqrt{2}} \qquad \sec\left(\frac{\pi}{4}\right) = \frac{1}{\cos\left(\frac{\pi}{4}\right)} = \sqrt{2}$$
$$\tan\left(\frac{\pi}{4}\right) = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{1} = 1 \qquad \cot\left(\frac{\pi}{4}\right) = \frac{1}{\tan\left(\frac{\pi}{4}\right)} = 1$$

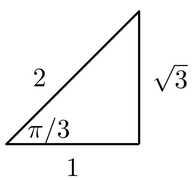
A 30-60-90 Triangle

Consider the equilateral triangle given below. Geometry allows us to construct a 30-60-90 triangle:



We can now determine the six trigonometric functions at two more angles!

$60^\circ = \dfrac{\pi}{3}$ radians:

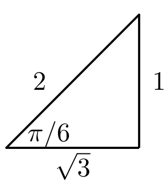


$$\sin\left(\frac{\pi}{3}\right) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2} \qquad \csc\left(\frac{\pi}{3}\right) = \frac{1}{\sin\left(\frac{\pi}{3}\right)} = \frac{2}{\sqrt{3}}$$

$$\cos\left(\frac{\pi}{3}\right) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{1}{2} \qquad \sec\left(\frac{\pi}{3}\right) = \frac{1}{\cos\left(\frac{\pi}{3}\right)} = 2$$

$$\tan\left(\frac{\pi}{3}\right) = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sqrt{3}}{1} = \sqrt{3} \qquad \cot\left(\frac{\pi}{3}\right) = \frac{1}{\tan\left(\frac{\pi}{3}\right)} = \frac{1}{\sqrt{3}}$$

$30^\circ = \dfrac{\pi}{6}$ radians:

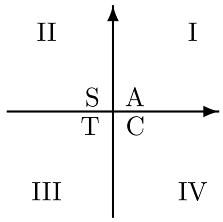


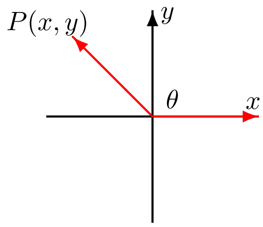
$$\sin\left(\frac{\pi}{6}\right) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{1}{2} \qquad \csc\left(\frac{\pi}{6}\right) = \frac{1}{\sin\left(\frac{\pi}{3}\right)} = 2$$

$$\cos\left(\frac{\pi}{6}\right) = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2} \qquad \sec\left(\frac{\pi}{6}\right) = \frac{1}{\cos\left(\frac{\pi}{3}\right)} = \frac{2}{\sqrt{3}}$$

$$\tan\left(\frac{\pi}{6}\right) = \frac{\text{opposite}}{\text{adjacent}} = \frac{1}{\sqrt{3}} \qquad \cot\left(\frac{\pi}{6}\right) = \frac{1}{\tan\left(\frac{\pi}{3}\right)} = \frac{\sqrt{3}}{1} = \sqrt{3}$$

- initial side, terminal side, vertex, standard position, coterminal angles, quadrantal angles, quadrants, CAST (4.3)



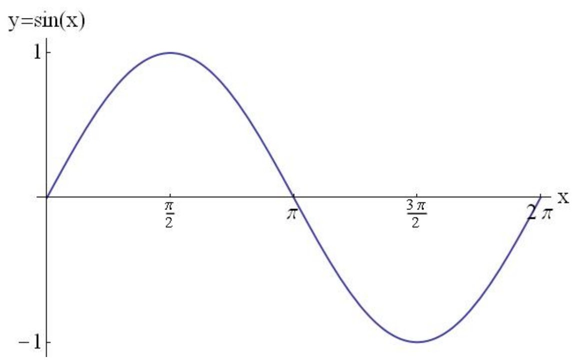


  If we label the point at the end of the terminal side as $P(x,y)$, and if we let $r=\sqrt{x^2+y^2}$, we can construct the following relationships between the six trig functions and our diagram:

$$\cos\theta=\frac{x}{r},\qquad \sin\theta=\frac{y}{r},\qquad \tan\theta=\frac{y}{x},\ x\neq 0$$
$$\csc\theta=\frac{r}{y},\ y\neq 0,\qquad \sec\theta=\frac{r}{x},\ x\neq 0,\qquad \cot\theta=\frac{x}{y},\ y\neq 0$$

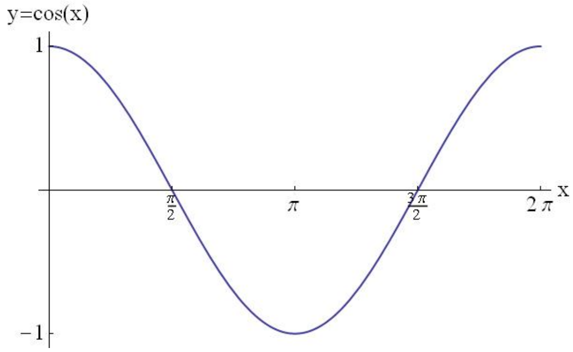
- the unit circle: the above relations with $r=1$ produces the unit circle. The coordinates around the unit circle satisfy $(x,y)=(\cos\theta,\sin\theta)$.
- a periodic function f satisfies $f(x)=f(x+c)$ where c is the smallest such number and c is called the period.
- graphs of sine, cosine, tangent, cotangent, secant, and cosecant
- sinusoids: period, frequency, phase shift, amplitude.
- inverse trigonometric functions
  $\arcsin x=\sin^{-1}x\neq(\sin x)^{-1}=\dfrac{1}{\sin x}$ and similarly for $\arccos x$ and $\arctan x$.

The Sine Function $\sin x$



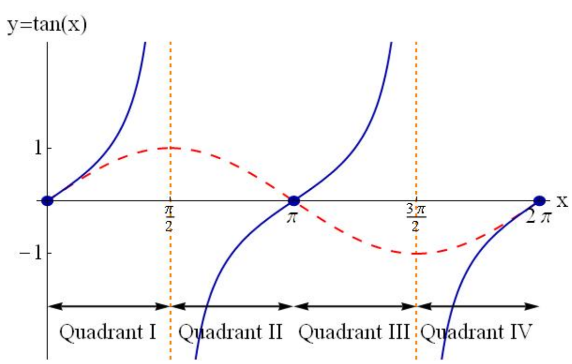
Domain: $x\in\mathbb{R}$
Range: $y\in[-1,1]$
Continuity: continuous for all x
Increasing-decreasing behaviour: alternately increasing and decreasing
Symmetry: odd ($\sin(-x)=-\sin(x)$))
Boundedness: bounded above and below
Local Extrema: absolute max of $y=1$, absolute min of $y=-1$
Horizontal Asymptotes: none
Vertical Asymptotes: none
End behaviour: The limits as x approaches $\pm\infty$ do not exist since the function values oscillate between $+1$ and -1.
This is a periodic function with period 2π.

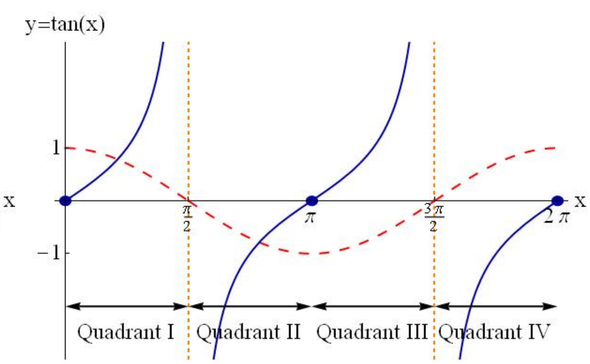
The Cosine Function $\cos x$



Domain: $x \in \mathbb{R}$
Range: $y \in [-1, 1]$
Continuity: continuous for all x
Increasing-decreasing behaviour: alternately increasing and decreasing
Symmetry: even $(\cos(-x) = \cos(x)))$
Boundedness: bounded above and below
Local Extrema: absolute max of $y = 1$, absolute min of $y = -1$
Horizontal Asymptotes: none
Vertical Asymptotes: none
End behaviour: The limits as x approaches $\pm\infty$ do not exist since the function values oscillate between $+1$ and -1.
This is a periodic function with period 2π.

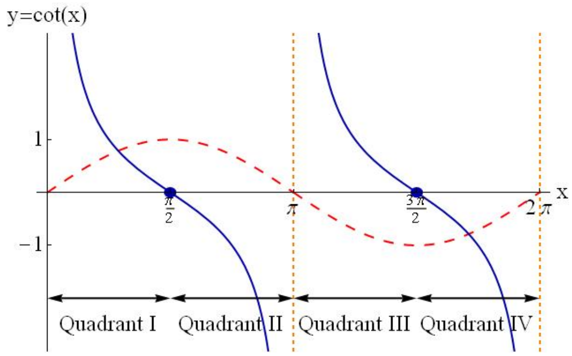
The Tangent Function $\tan x = \frac{\sin x}{\cos x}$

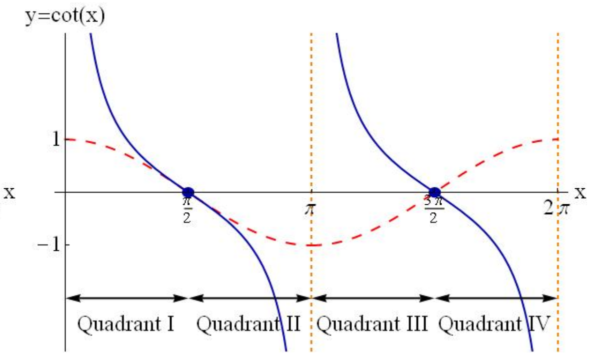




Domain: $x \in \mathbb{R}$ except $x = \dfrac{\pi}{2} + k\pi,\ k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
Range: $y \in \mathbb{R}$
Continuity: continuous on its domain
Increasing-decreasing behaviour: increasing on each interval in its domain
Symmetry: odd $(\tan(-x) = -\tan(x)))$
Boundedness: not bounded
Local Extrema: none
Horizontal Asymptotes: none
Vertical Asymptotes: $x = \dfrac{\pi}{2} + k\pi,\ k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
End behaviour: The limits as x approaches $\pm\infty$ do not exist since the function values oscillate between $-\infty$ and $+\infty$.
This is a periodic function with period π.

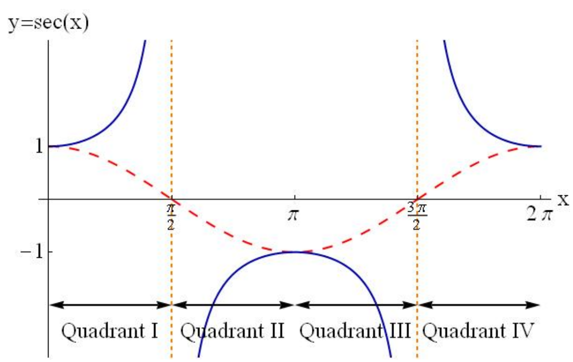
The Cotangent Function $\cot x = \frac{\cos x}{\sin x}$





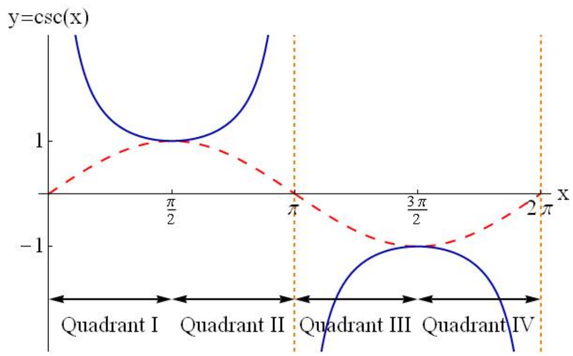
Domain: $x \in \mathbb{R}$ except $x = k\pi,\ k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
Range: $y \in \mathbb{R}$
Continuity: continuous on its domain
Increasing-decreasing behaviour: decreasing on each interval in its domain
Symmetry: odd $(\cot(-x) = -\cot(x))$
Boundedness: not bounded
Local Extrema: none
Horizontal Asymptotes: none
Vertical Asymptotes: $x = k\pi,\ k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
End behaviour: The limits as x approaches $\pm\infty$ do not exist since the function values oscillate between $-\infty$ and $+\infty$.
This is a periodic function with period π.

The Secant Function $\sec x = \frac{1}{\cos x}$



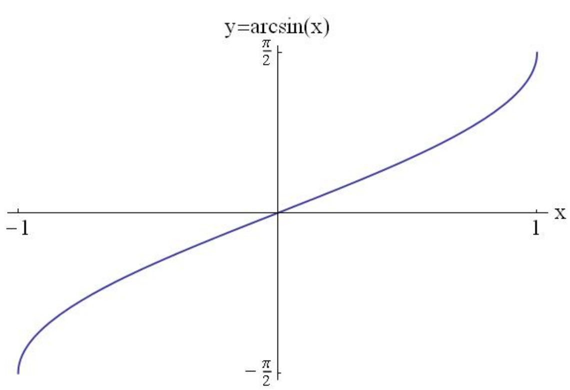
Domain: $x \in \mathbb{R}$ except $x = \dfrac{\pi}{2} + k\pi,\ k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
Range: $y \in (-\infty, -1] \cup [1, \infty)$
Continuity: continuous on its domain
Increasing-decreasing behaviour: increases and decreases on each interval in its domain
Symmetry: even $(\sec(-x) = \sec(x))$
Boundedness: not bounded
Local Extrema: local min at $x = 2k\pi$, local max at $x = (2k+1)\pi,\ k = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$
Horizontal Asymptotes: none
Vertical Asymptotes: $x = \dfrac{\pi}{2} + k\pi,\ k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
End behaviour: The limits as x approaches $\pm\infty$ do not exist since the function values oscillate between $-\infty$ and $+\infty$.
This is a periodic function with period 2π.

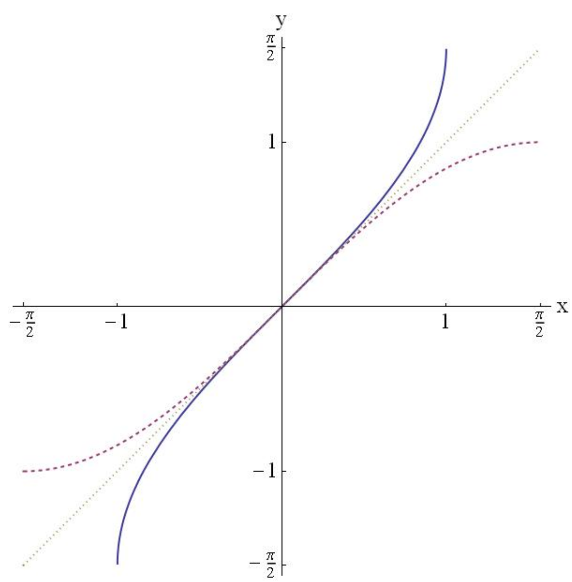
The Cosecant Function $\csc x = \frac{1}{\sin x}$



Domain: $x \in \mathbb{R}$ except $x = k\pi$, $k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
Range: $y \in (-\infty, -1] \cup [1, \infty)$
Continuity: continuous on its domain
Increasing-decreasing behaviour: increases and decreases on each interval in its domain
Symmetry: odd ($\csc(-x) = -\csc(x)$))
Boundedness: not bounded
Local Extrema: local min at $x = \pi/2 + 2k\pi$, local max at $x = 3\pi/2 + 2k\pi$, $k = \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots$
Horizontal Asymptotes: none
Vertical Asymptotes: $x = k\pi$, $k = \ldots, -3, 2, 1, 0, 1, 2, 3, \ldots$
End behaviour: The limits as x approaches $\pm\infty$ do not exist since the function values oscillate between $-\infty$ and $+\infty$.
This is a periodic function with period 2π.

The Inverse Sine Function $\arcsin x$

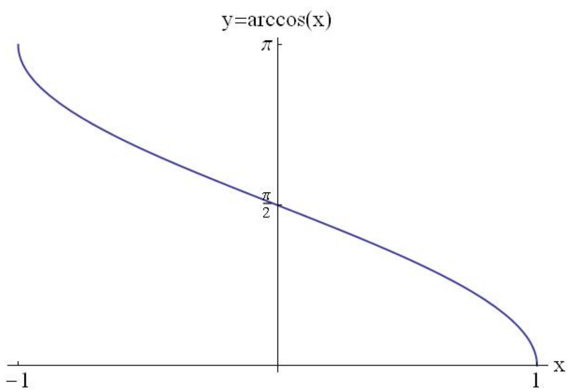


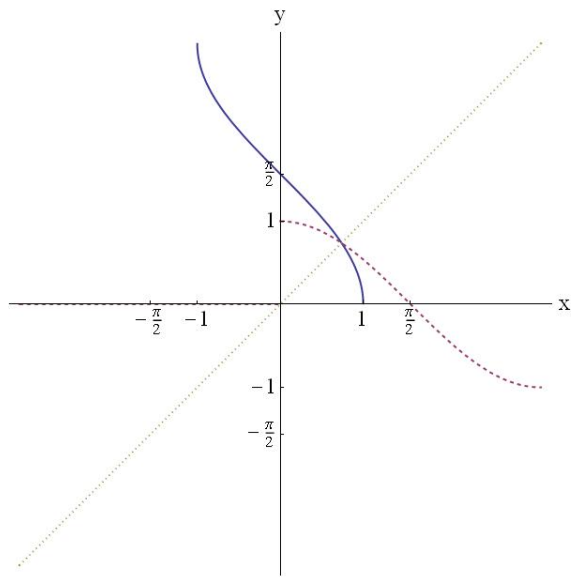


Domain: $x \in [-1, 1]$
Range: $y \in [-\pi/2, \pi/2]$
Continuity: continuous for all x in domain
Increasing-decreasing behaviour: increasing
Symmetry: odd ($\arcsin(-x) = -\arcsin(x)$))

Boundedness: bounded above and below
Local Extrema: absolute max of $y = \pi/2$, absolute min of $y = -\pi/2$
Horizontal Asymptotes: none
Vertical Asymptotes: none
End behaviour: The limits as x approaches $\pm\infty$ do not exist.

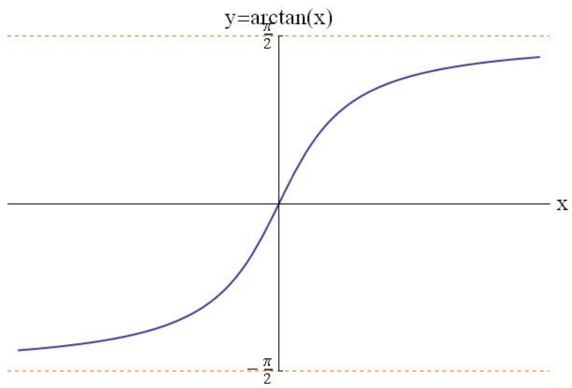
The Inverse Cosine Function $\arccos x$

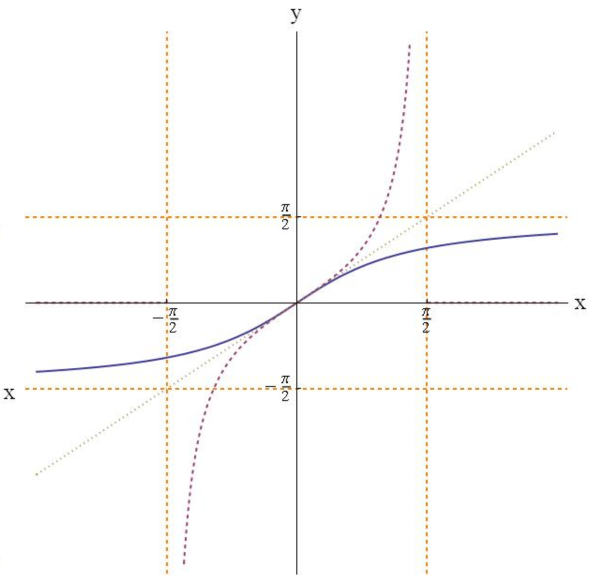




Domain: $x \in [-1, 1]$
Range: $y \in [0, \pi]$
Continuity: continuous for all x in domain
Increasing-decreasing behaviour: decreasing
Symmetry: none
Boundedness: bounded above and below
Local Extrema: absolute max of $y = \pi$, absolute min of $y = 0$
Horizontal Asymptotes: none
Vertical Asymptotes: none
End behaviour: The limits as x approaches $\pm\infty$ do not exist.

The Inverse Tangent Function $\arctan x$





Domain: $x \in \mathbb{R}$
Range: $y \in (-\pi/2, \pi/2)$
Continuity: continuous for all x
Increasing-decreasing behaviour: increasing
Symmetry: odd $(\arctan(-x) = -\arctan(x))$
Boundedness: bounded above and below
Local Extrema: absolute max of $y = \pi/2$, absolute min of $y = -\pi/2$
Horizontal Asymptotes: $y = \pm\pi/2$
Vertical Asymptotes: none
End behaviour: $\lim_{x\to\infty} \arctan x = \frac{\pi}{2}$ $\quad$ $\lim_{x\to-\infty} \arctan x = -\frac{\pi}{2}$