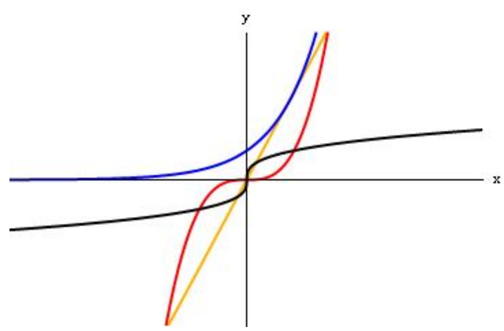
Questions

Example Classify each function as a power function, root function, polynomial (state its degree), rational function, algebraic function, trigonometric function, exponential function, or logarithmic function.

a) $y = \dfrac{x-6}{x+6}$ b) $y = x + \dfrac{x^2}{\sqrt{x-1}}$
c) $y = 10^x$ d) $y = x^{10}$
e) $y = 2t^6 + t^4 - \pi$ f) $y = \cos\theta + \sin\theta$

Example Match each equation with its graph. Explain your choices.

a) $y = 3x$ b) $y = 3^x$ c) $y = x^3$ d) $y = x^{1/3}$



Example At the surface of the ocean, the water pressure is the same as the air pressure above the water, 15 lb/in^2. Below the surface, the water pressure increases by 4.43 lb/in^2 for every 10 ft of descent.

a) Express the water pressure as a function of the depth below the ocean surface.
b) At what depth is the pressure 100 lb/in^2?

Example The table gives the winning heights for the (men's) Olympic pole vault competition in the 20th century.

Year	Height (ft)	Year	Height (ft)
1900	10.83	1956	14.96
1904	11.48	1960	15.42
1908	12.17	1964	16.73
1912	12.96	1968	17.71
1920	13.42	1972	18.04
1924	12.96	1976	18.04
1928	13.77	1980	18.96
1932	14.15	1984	18.85
1936	14.27	1988	19.77
1948	14.10	1992	19.02
1952	14.92	1996	19.42

a) Make a scatter plot and decide whether a linear model is appropriate.
b) Find and graph the regression line.
c) Use the linear model to predict the height of the winning pole vault at the 2000 Olympics.
d) Is it reasonable to use the model to predict the winning height at the 2100 Olympics?

You can do this by hand, or use the Mathematica commands `ListPlot` *and* `Fit`.

Solutions

Example Classify each function as a power function, root function, polynomial (state its degree), rational function, algebraic function, trigonometric function, exponential function, or logarithmic function.

a) $y = \dfrac{x-6}{x+6}$ b) $y = x + \dfrac{x^2}{\sqrt{x-1}}$
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e) $y = 2t^6 + t^4 - \pi$ f) $y = \cos\theta + \sin\theta$

a) Rational function, since it is a ratio of polynomials.
b) Algebraic function, since it is constructed using algebraic operations (which includes roots!) on polynomials.
c) Exponential function, since the base (10) is a positive constant.
d) Power function, since the base (x) is raised to a constant.
e) Polynomial function of degree 6.
f) Trigonometric function, since it involves trig functions.

Example Match each equation with its graph. Explain your choices.

Let's begin by separating out the graphs, and then analysing.

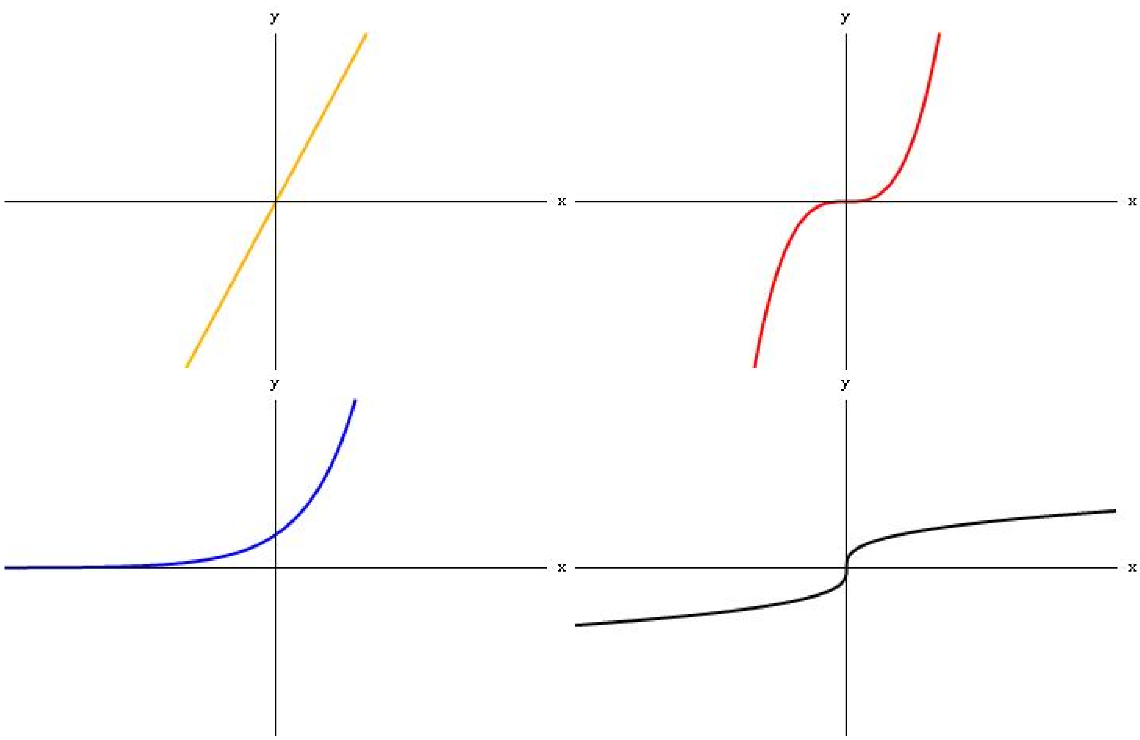

Figure 1: The graphs.

The graph on the upper left is linear in x, so it must be the function $y = 3x$.

The graph on the lower left is representative of exponential growth, so it must go with the function $y = 3^x$. Also, it is the only function of the four given which does not go through the origin.

The two graphs on the right look similar (both odd). We have to decide which is $y = x^3$ and which is $y = x^{1/3}$. Since $1/3 < 3$, we can say that $y = x^{1/3}$ will not grow as rapidly as $y = x^3$ for large values of x. From this information, we can say that the graph on the upper right is $y = x^3$ and on the lower right is $y = x^{1/3}$.

Example At the surface of the ocean, the water pressure is the same as the air pressure above the water, 15 lb/in^2. Below the surface, the water pressure increases by 4.43 lb/in^2 for every 10 ft of descent.

a) Express the water pressure as a function of the depth below the ocean surface.
b) At what depth is the pressure 100 lb/in^2?

First, we need to say something about the mathematical labels we will be using. Let

d	be the depth in the ocean in ft. We must have $d \geq 0$.
P	be the water pressure in lb/in^2 at a depth of d ft.

We are told that water pressure increases by 4.34 lb/in^2 for every 10 ft of descent. This is telling us that their is a *linear relationship* between depth and water pressure.

So we know that $P(d) = md + b$, where m is the rate of change (slope) of water pressure with respect to depth and b is the P-intercept. The P-intercept is the value when $d = 0$, which we are told is 15 lb/in^2. The slope is given by 4.34/10 = 0.434 lb/in^2 (it is the rate of change of water pressure with respect to depth).

Therefore, the equation for water pressure in ln/in^2 as a function of ocean depth in ft is

$$P(d) = 0.434d + 15.$$

Note that the domain for this function is $d \geq 0$, which is imposed by the mathematical model, not the mathematics!

To find when the pressure is 100 lb/in^2, we must solve the equation $P(d) = 100 = 0.434d + 15$ for d. Doing this, we find that the pressure is 100 lb/in^2 at a depth of 196 ft.

Example The table gives the winning heights for the (men's) Olympic pole vault competition in the 20th century.

Year	Height (ft)	Year	Height (ft)
1900	10.83	1956	14.96
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a) Make a scatter plot and decide whether a linear model is appropriate.
b) Find and graph the regression line.
c) Use the linear model to predict the height of the winning pole vault at the 2000 Olympics.
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I created this graph using *Mathematica*, which you will learn how to do soon enough! For now, you can create the graph by hand.

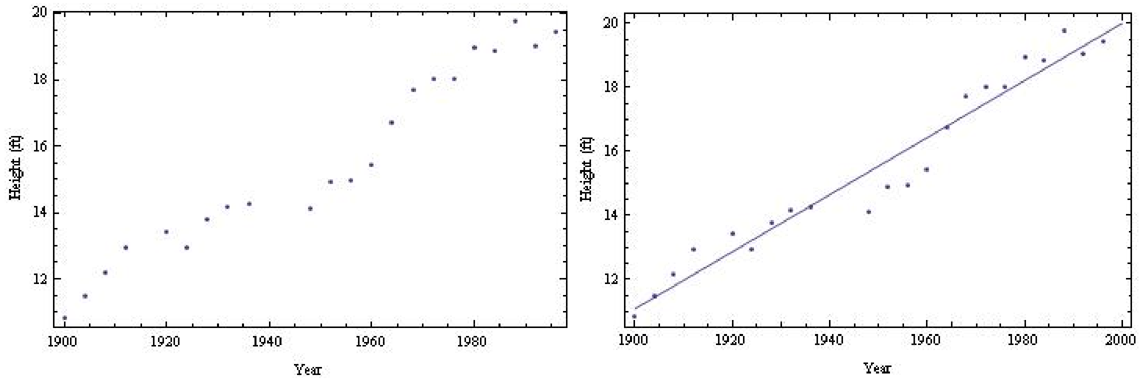

Figure 2: The graph of Olympic pole vaulting results. The graph on the left also contains the linear fit $y = 0.089x - 158$.

Although there is a great deal of oscillatory variation in the graph, it would be appropriate to get rough estimates of values by approximating the data using a linear model. For now, you can simply draw a best fit line through the data and determine the equation of the line using a point on the line (x_0, y_0) and the slope of the line and the equation $y - y_0 = m(x - x_0)$ (point-slope equation of a line–see Appendix B). In statistics and other math courses you will learn more about how to better fit a curve to a set of data points.

Using the linear fit, we can predict the winning pole vault in the 2000 Olympics to be $y = 0.089(2000) - 158 = 20$ ft (the winning result was 5.9 m $\sim$ 19.36 ft).

Using the linear fit to predict a result in 2100 would not be appropriate, since we would be extrapolating a result very far from the data. We would also need to use our knowledge of pole vaulting to tell us that improvement at the level the model predicts is unlikely. Both the mathematics and the physical system we are modeling can help us guide our efforts to create and evaluate models.