

10.1.31 | a) The parametric equation

$$\begin{aligned}x &= x_1 + (x_2 - x_1)t \\ y &= y_1 + (y_2 - y_1)t\end{aligned} \quad 0 \leq t \leq 1$$

includes the point (x_1, y_1) when $t=0$, and (x_2, y_2) when $t=1$. So these are the endpoints of the curve. Now we just need to show that the rest of the points on the curve are the straight line segment between these two points. We can do that by eliminating the t and getting an implicit representation:

$$\begin{aligned}x &= x_1 + (x_2 - x_1)t \longrightarrow t = \frac{x - x_1}{x_2 - x_1} \\ y &= y_1 + (y_2 - y_1)t \longrightarrow t = \frac{y - y_1}{y_2 - y_1}\end{aligned} \implies \frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

Since $\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$ is the formula for the straight line between the points (x_1, y_1) and (x_2, y_2) , we've answered part a).

b)

$$\begin{aligned}x &= -2 + (3 - (-2))t = -2 + 5t \\ y &= 7 + (-1 - 7)t = 7 - 8t\end{aligned} \quad 0 \leq t \leq 1.$$