Questions

Example Sketch the graph and give an example of a function that satisfies all of the given conditions.

$\lim_{x\to 0^-} f(x) = 1,\ \lim_{x\to 0^+} f(x) = -1,\ \lim_{x\to 2^-} f(x) = 0,\ \lim_{x\to 2^+} f(x) = 1,\ f(2) = 1,\ f(0)$ is undefined.

Example Estimate the value of the limit $\lim_{x\to 0}(1+x)^{1/x}$ to five decimal places. Does this number look familiar? Illustrate by graphing the function $y = (1+x)^{1/x}$.

Solutions

Example Sketch the graph and give an example of a function that satisfies all of the given conditions.

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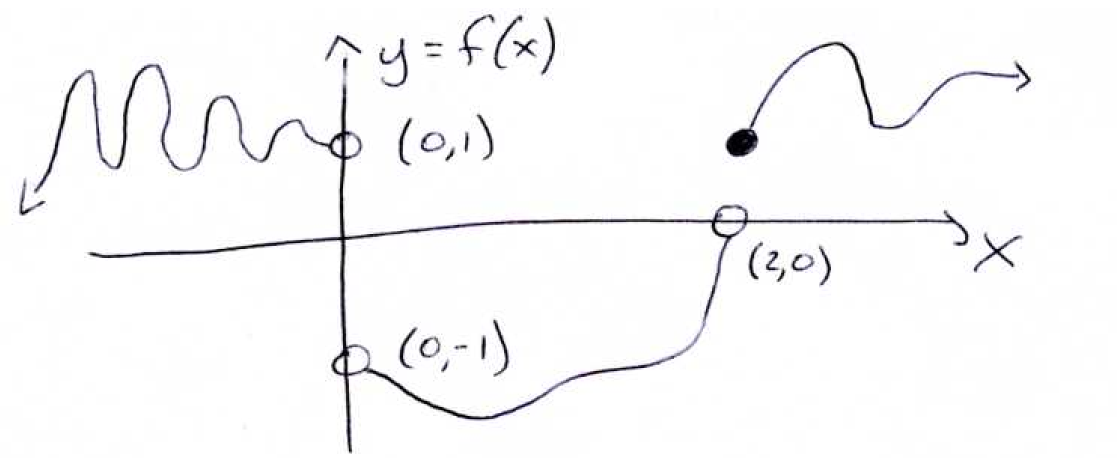
You can construct many different graphs that have the properties listed. One is the following:



Example Estimate the value of the limit $\lim_{x\to 0}(1+x)^{1/x}$ to five decimal places. Does this number look familiar? Illustrate by graphing the function $y = (1+x)^{1/x}$.

To estimate the limit $\lim_{x\to 0}(1+x)^{1/x}$ to five decimals, we can evaluate the function at numbers closer and closer to zero and create a table. We could also create graph using *Mathematica*, and zoom in closer and closer to zero. I chose to do the latter. The command you can use to do this is

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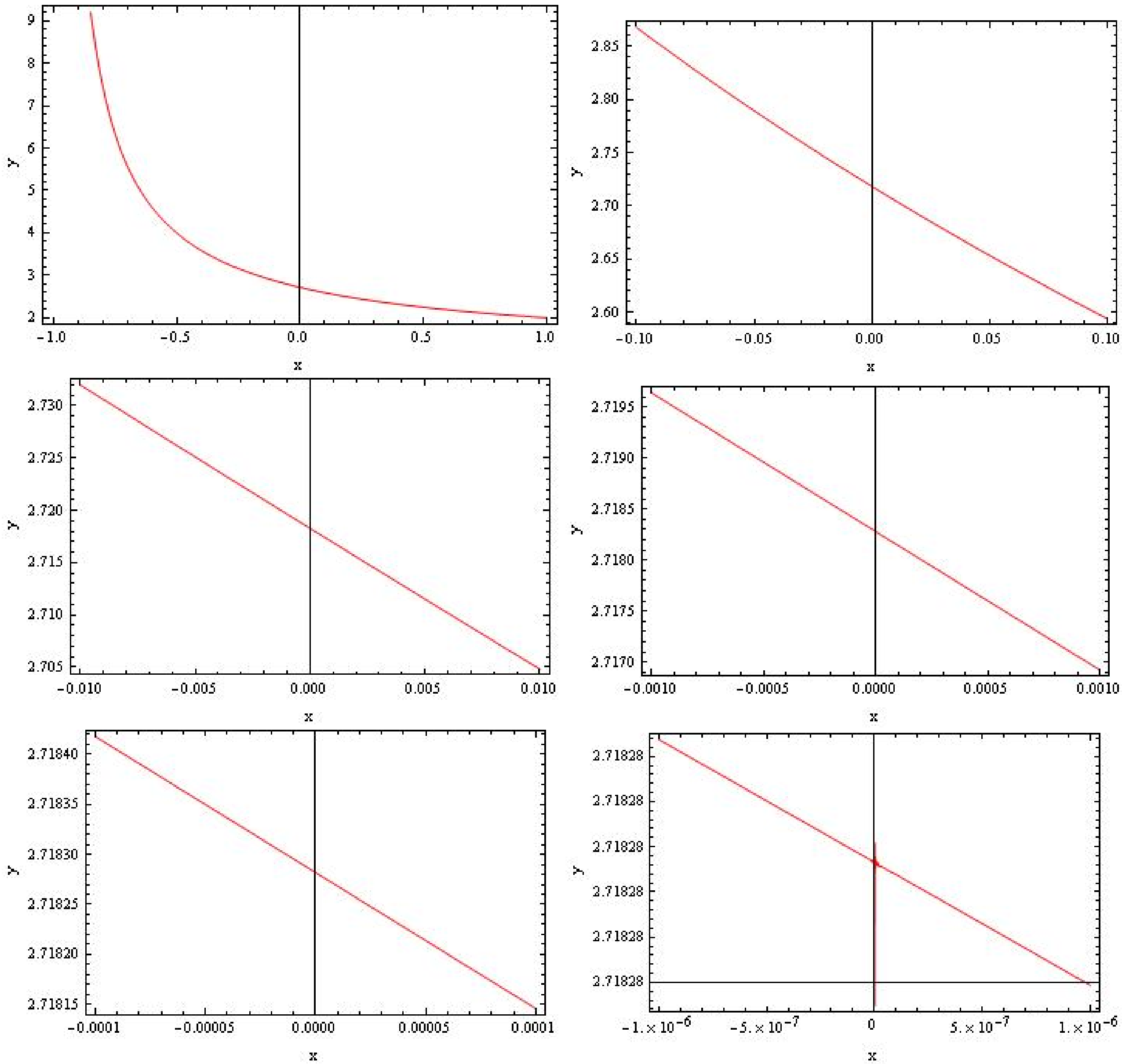
Plot[{(1 + x)^(1/x)}, {x, -0.01, 0.01}]
```



From the last graph I estimate $\lim_{x\to 0}(1+x)^{1/x} \sim 2.71828$. Notice that the graph is getting messy due to numerical round off problems. The graph should be looking more and more like a straight line as we zoom in, however, numerical errors creep in and cause problems when we try to evaluate a number very very close to one raised to a power which is very very large. Try zooming in even more and see what kind of a mess you can make. This is why it is so important to have analytic methods to determine limits. We cannot rely on numerical methods to be accurate.

This number (2.71828) sure reminds me of the number e we saw in Section 1.5! It looks like a transcendental number can be expressed in terms of a limit. How cool is that? *Very!*