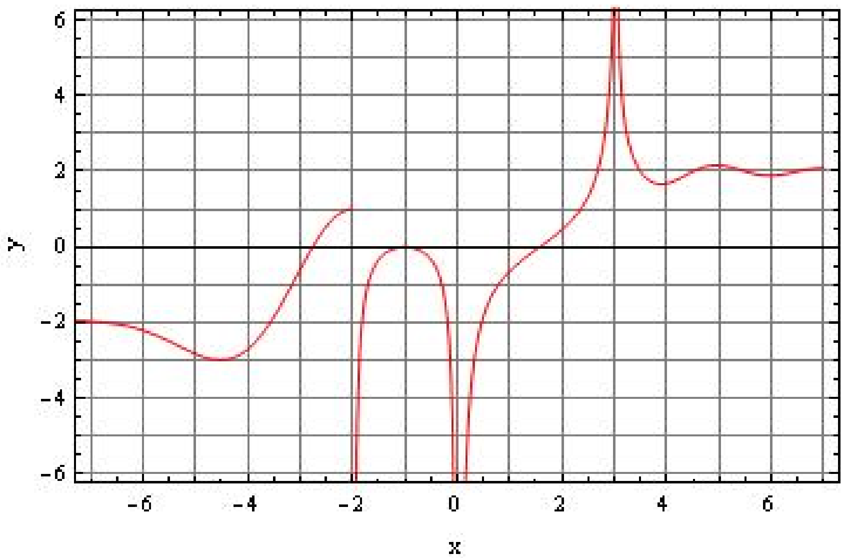
Questions

Example For the function g whose graph is given, state the following

a) $\lim_{x\to\infty} g(x)$ b) $\lim_{x\to-\infty} g(x)$ c) $\lim_{x\to 3} g(x)$ d) $\lim_{x\to 0} g(x)$
e) $\lim_{x\to 2^+} g(x)$ f) equations of the asymptotes.



Example Sketch the graph of an example of a function f that satisfies all of the following conditions:

a) $\lim_{x\to 2} f(x) = -\infty$ b) $\lim_{x\to\infty} f(x) = \infty$ c) $\lim_{x\to-\infty} f(x) = 0$
d) $\lim_{x\to 0^+} f(x) = \infty$ e) $\lim_{x\to 0^-} f(x) = -\infty$

Example Find the limit $\lim_{x\to-\infty} \dfrac{\sqrt{9x^6 - x}}{x^3+1}$.

Example Evaluate the limit and justify each step by indicating the appropriate properties of limits.

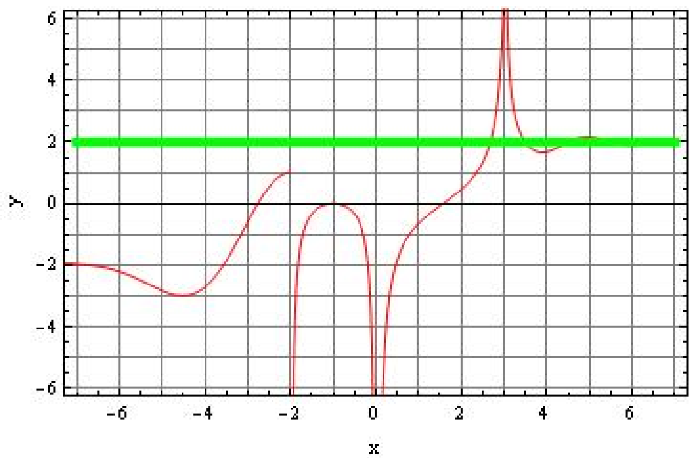
$$\lim_{x\to-\infty} \frac{(1-x)(2+x)}{(1+2x)(2-3x)}.$$

Solutions

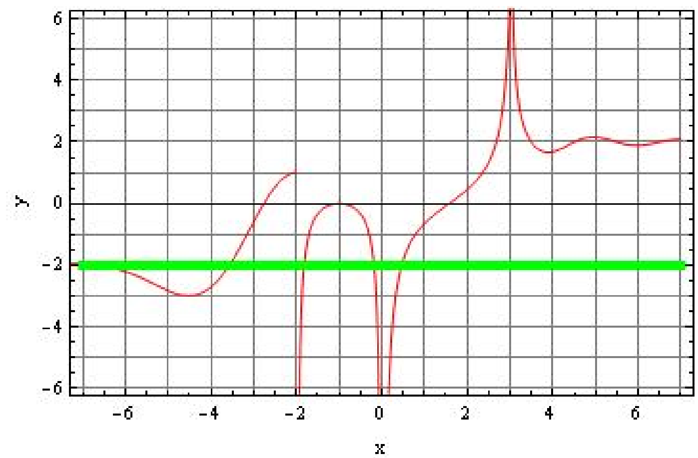
Example For the function g whose graph is given, state the following

a) $\lim_{x\to\infty} g(x)$ b) $\lim_{x\to-\infty} g(x)$ c) $\lim_{x\to 3} g(x)$ d) $\lim_{x\to 0} g(x)$
e) $\lim_{x\to 2^+} g(x)$ f) equations of the asymptotes.

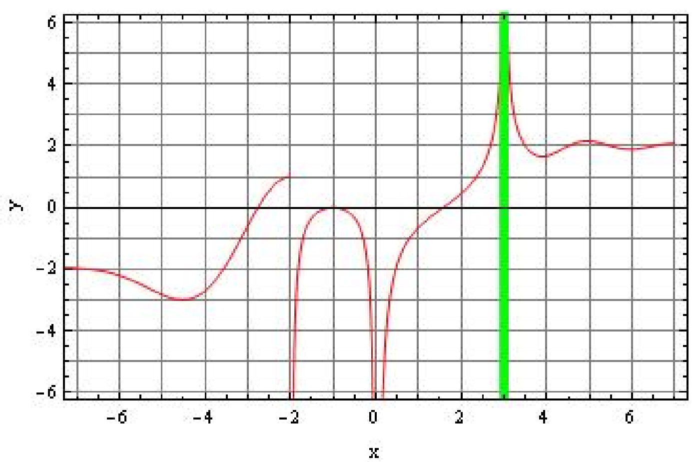
Here, I will reproduce the graph with a green line indicating where it is on the graph we are looking to evaluate the limit. The red line should be approaching the green line (or dot!) as we approach the appropriate value of x. The green lines (but not the dot!) are asymptotes, and their equations are also given.



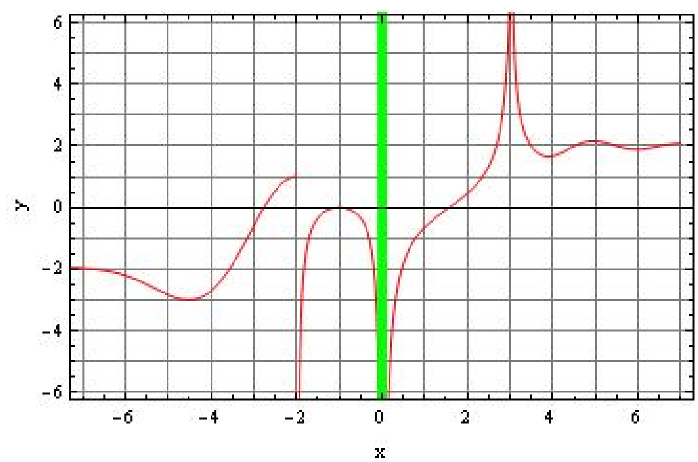
$$\lim_{x \to \infty} g(x) = 2$$
Horizontal Asymptote: $y = 2$



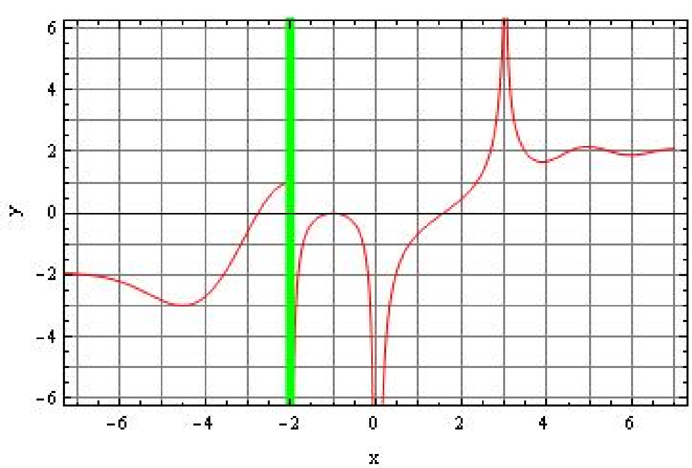
$$\lim_{x \to -\infty} g(x) = -2$$
Horizontal Asymptote: $y = -2$



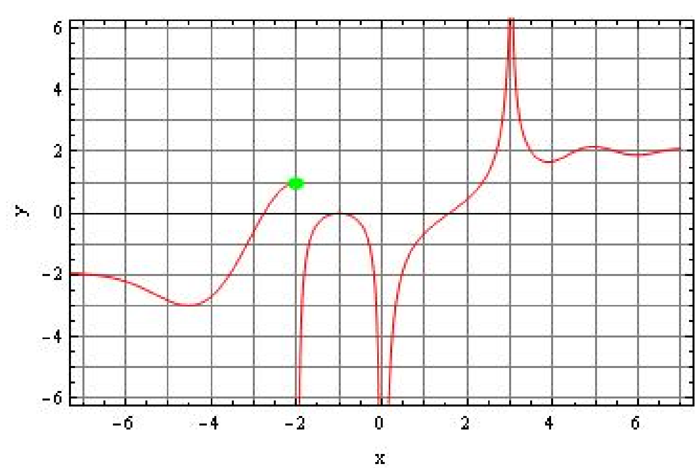
$$\lim_{x \to 3} g(x) = \infty \text{ (Note: both sides)}$$
Vertical Asymptote: $x = 3$



$$\lim_{x \to 0} g(x) = -\infty \text{ (Note: both sides)}$$
Vertical Asymptote: $x = 0$



$$\lim_{x \to 2^+} g(x) = -\infty$$
Vertical Asymptote: $x = -2$

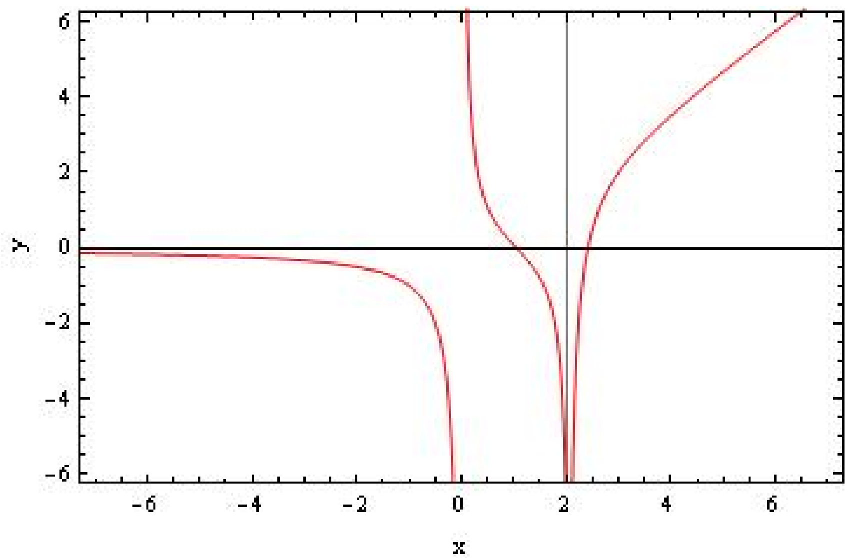


$$\lim_{x \to 2^-} g(x) = 1$$

Example Sketch the graph of an example of a function f that satisfies all of the following conditions:

a) $\lim\limits_{x\to 2} f(x) = -\infty$ b) $\lim\limits_{x\to\infty} f(x) = \infty$ c) $\lim\limits_{x\to -\infty} f(x) = 0$
d) $\lim\limits_{x\to 0^+} f(x) = \infty$ e) $\lim\limits_{x\to 0^-} f(x) = -\infty$

There are literally an infinite number of functions which will satisfy the requirements. I provide one below.



Example Find the limit $\lim\limits_{x\to -\infty} \dfrac{\sqrt{9x^6 - x}}{x^3+1}$.

$$\begin{aligned}
\lim_{x\to -\infty} \frac{\sqrt{9x^6 - x}}{x^3+1} &= \lim_{x\to -\infty} \frac{\sqrt{9x^6 - x}\cdot\left(\frac{1}{x^3}\right)}{(x^3+1)\cdot\left(\frac{1}{x^3}\right)} \\
&= \lim_{x\to -\infty} \frac{\sqrt{9x^6 - x}\cdot\left(\frac{-1}{\sqrt{(x^3)^2}}\right)}{1+\frac{1}{x^3}} \text{ since } x<0 \\
&= \lim_{x\to -\infty} \frac{-\sqrt{\frac{9x^6-x}{x^6}}}{1+\frac{1}{x^3}} \\
&= \lim_{x\to -\infty} \frac{-\sqrt{9-\frac{1}{x^5}}}{1+\frac{1}{x^3}} \\
&= \frac{-\sqrt{9-0}}{1+0} \\
&= -3
\end{aligned}$$

Example Evaluate the limit and justify each step by indicating the appropriate properties of limits.

$$\lim_{x\to -\infty} \frac{(1-x)(2+x)}{(1+2x)(2-3x)}.$$

Here we just need to be explicit in the steps we use to solve the problem.

$$
\begin{aligned}
\lim_{x\to-\infty}\frac{(1-x)(2+x)}{(1+2x)(2-3x)} &= \lim_{x\to-\infty}\frac{2-x-x^2}{2+x-6x^2} \quad \text{(multiply out)} \\
&= \lim_{x\to-\infty}\frac{\frac{2}{x^2}-\frac{x}{x^2}-\frac{x^2}{x^2}}{\frac{2}{x^2}+\frac{x}{x^2}-\frac{6x^2}{x^2}} \quad \text{(divide all by } x^2\text{, highest power of } x \text{ in denominator)} \\
&= \lim_{x\to-\infty}\frac{\frac{2}{x^2}-\frac{1}{x}-1}{\frac{2}{x^2}+\frac{1}{x}-6} \quad \text{(simplify)} \\
&= \frac{\lim_{x\to-\infty}\left(\frac{2}{x^2}-\frac{1}{x}-1\right)}{\lim_{x\to-\infty}\left(\frac{2}{x^2}+\frac{1}{x}-6\right)} \quad \text{(law 5)} \\
&= \frac{2\lim_{x\to-\infty}\frac{1}{x^2}-\lim_{x\to-\infty}\frac{1}{x}-\lim_{x\to-\infty}1}{2\lim_{x\to-\infty}\frac{1}{x^2}+\lim_{x\to-\infty}\frac{1}{x}-\lim_{x\to-\infty}6} \quad \text{(law 1, law 2, law 3)} \\
&= \frac{2(0)-(0)-1}{2(0)+(0)-6} \quad \text{(law 7)} \\
&= \frac{-1}{-6}=\frac{1}{6} \quad \text{(simplify)}
\end{aligned}
$$