Questions

Example Graph the curve $y = e^x$ in the viewing rectangles $[-1, 1]$ by $[0, 2]$, $[-0.5, 0.5]$ by $[0.5, 1.5]$, and $[-0.1, 0.1]$ by $[0.9, 1.1]$. What do you notice about the curve as you zoom in toward the point $(0, 1)$?

Example Find an equation of the tangent line to the curve at the given point.

$$y = \frac{x-1}{x-2}, \quad (3, 2).$$

Example Find an equation of the tangent line to the curve at the given point.

$$y = 1/x^2, \quad (-2, 1/4).$$

Example Find the slope of the tangent to the curve $y = x^3 - 4x + 1$ at the point where $x = a$. Find equations of the tangent lines at the points $(1, -2)$ and $(2, 1)$. Graph the curve and both tangents.

Example Sketch the graph of a function g for which $g(0) = 0$, $g'(0) = 3$, $g'(1) = 0$, and $g'(2) = 1$.

Example If $g(x) = 1 - x^3$, find $g'(0)$ and use it to find an equation of the tangent line to the curve $y = 1 - x^3$ at the point $(0, 1)$.

Example Determine whether or not $f'(0)$ exists.

$$f(x) = \begin{cases} x \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Example Find the derivative of $f(x) = x^2 - 8x + 9$ at $x = a$.

Example Find an equation of the tangent line to the parabola $y = x^2 - 8x + 9$ at the point $(5, -6)$.

Example The position of a particle is given by the equation of motion $s = f(t) = 1/(1 + t)$, where t is in seconds and s is in meters. Find the velocity and speed of the particle at $t = 2$ seconds.

Example Find $f'(a)$ if $f(x) = \sqrt{3x + 1}$.

Example A particle moves along a straight line with equation of motion $s = f(t) = 2t^3 - t$, where s is measured in meters and t in seconds. Find the velocity when $t = 2$.

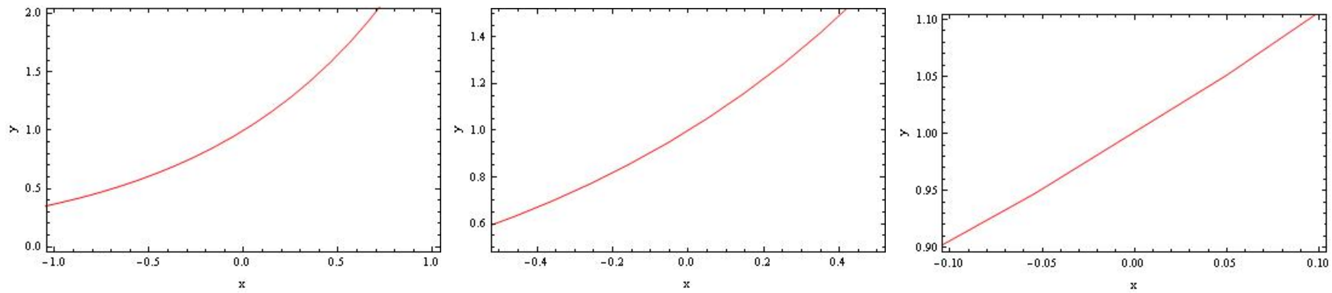
Example Find an equation of the tangent line to the function $y = 5/(x - 2)$ at the point $(1, -5)$.

Example The position of a particle is given by $s(t) = \sqrt{t^2 + 1}$ where s is measured in meters and t is measured in seconds. What is the instantaneous velocity of the particle when $t = 1$ second?

Solutions

Example Graph the curve $y = e^x$ in the viewing rectangles $[-1, 1]$ by $[0, 2]$, $[-0.5, 0.5]$ by $[0.5, 1.5]$, and $[-0.1, 0.1]$ by $[0.9, 1.1]$. What do you notice about the curve as you zoom in toward the point $(0, 1)$?

The three graphs are below. I notice that as you zoom in, the graph looks more like a straight line.



The *Mathematica* commands I used to create the graphs are listed below. I expect you to be able to define functions and plot with *Mathematica*, so the extra options I am using are strictly for your information, not the sort of thing I would expect you to use when you use *Mathematica* to solve problems.

```
plot = Plot[{Exp[x]}, {x, -5, 5}, PlotStyle -> {Red}, PlotPoints -> 100]
plot1 = Show[plot, Axes -> False, Frame -> True, FrameLabel -> {"x", "y"},
    PlotRange -> {{-1, 1}, {0, 2}}]
plot2 = Show[plot, Axes -> False, Frame -> True, FrameLabel -> {"x", "y"},
    PlotRange -> {{-0.5, 0.5}, {0.5, 1.5}}]
plot3 = Show[plot, Axes -> False, Frame -> True, FrameLabel -> {"x", "y"},
    PlotRange -> {{-0.1, 0.1}, {0.9, 1.1}}]
```

Example Find an equation of the tangent line to the curve at the given point.

$$y = \frac{x-1}{x-2}, \quad (3, 2).$$

To begin, let's talk in general before we get to the problem we are asked to solve.

The equation of the tangent line will be the equation of a straight line. The point-slope equation for a lines is

$$y - y_0 = m(x - x_0),$$

where the line passes through the point (x_0, y_0) and has slope m. The slope of the tangent line to the curve $y = f(x)$ is given by

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

We now have everything we need to solve the problem. Let's solve it!

$$
\begin{aligned}
f(x) &= \frac{x-1}{x-2} \\
f(a) &= \frac{a-1}{a-2} \\
f(a+h) &= \frac{a+h-1}{a+h-2} \\
m &= \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \\
&= \lim_{h\to 0} \frac{\frac{a+h-1}{a+h-2} - \frac{a-1}{a-2}}{h} \quad \text{Indeterminant quotient; simplify: common denominator} \\
&= \lim_{h\to 0} \frac{1}{h}\left[\frac{(a+h-1)(a-2)-(a-1)(a+h-2)}{(a+h-2)(a-2)}\right] \\
&= \lim_{h\to 0} \frac{1}{h}\left[\frac{(2-3a+a^2-2h+ah)-(2-3a+a^2-h+ah)}{(a+h-2)(a-2)}\right] \\
&= \lim_{h\to 0} \frac{1}{h}\left[\frac{2-3a+a^2-2h+ah-2+3a-a^2+h-ah}{(a+h-2)(a-2)}\right] \\
&= \lim_{h\to 0} \frac{1}{h}\left[\frac{-h}{(a+h-2)(a-2)}\right] \\
&= \lim_{h\to 0} \left[\frac{-1}{(a+h-2)(a-2)}\right] \\
&= \frac{-1}{(a+0-2)(a-2)} \\
&= \frac{-1}{(a-2)^2}
\end{aligned}
$$

This is the slope of the tangent line at $x = a$. The point the tangent line passes through is $(3, 2)$, which means $x_0 = 3$ and $y_0 = 2$. That also means that our slope should be evaluated at $a = x_0 = 3$, so

$$
\begin{aligned}
m &= \left.\frac{-1}{(a-2)^2}\right|_{a=3} \\
&= \frac{-1}{(3-2)^2} \\
&= -1
\end{aligned}
$$

The equation of the tangent line to the curve $f(x) = \frac{x-1}{x-2}$ at the point $(3, 2)$ is therefore:

$$y - y_0 = m(x - x_0) \quad \longrightarrow \quad y - 2 = (-1)(x-3) \quad \longrightarrow \quad y = -x + 5.$$

Example Find an equation of the tangent line to the curve at the given point.

$y = 1/x^2, \quad (-2, 1/4).$

To begin, let's talk in general before we get to the problem we are asked to solve.

The equation of the tangent line will be the equation of a straight line. The point-slope equation for a lines is

$y - y_0 = m(x - x_0),$

where the line passes through the point (x_0, y_0) and has slope m. The slope of the tangent line to the curve $y = f(x)$ is given by

$$m = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}.$$

We now have everything we need to solve the problem. Let's solve it!

$$\begin{aligned}
f(x) &= \frac{1}{x^2} \\
f(a) &= \frac{1}{a^2} \\
f(a+h) &= \frac{1}{(a+h)^2} \\
&= \frac{1}{a^2 + 2ah + h^2} \\
m &= \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} \\
&= \lim_{h\to 0} \frac{\left(\frac{1}{a^2+2ah+h^2}\right) - \left(\frac{1}{a^2}\right)}{h} \\
&= \lim_{h\to 0} \frac{1}{h}\left[\frac{1}{a^2+2ah+h^2} - \frac{1}{a^2}\right] \quad \text{Indeterminant quotient; simplify: common denominator} \\
&= \lim_{h\to 0} \frac{1}{h}\left[\frac{a^2}{a^2(a^2+2ah+h^2)} - \frac{a^2+2ah+h^2}{a^2(a^2+2ah+h^2)}\right] \\
&= \lim_{h\to 0} \frac{1}{h}\left[\frac{a^2 - a^2 - 2ah - h^2}{a^2(a^2+2ah+h^2)}\right] \\
&= \lim_{h\to 0} \frac{1}{h}\left[\frac{-2ah - h^2}{a^2(a^2+2ah+h^2)}\right] \\
&= \lim_{h\to 0} \frac{h}{h}\left[\frac{-2a - h}{a^2(a^2+2ah+h^2)}\right] \\
&= \lim_{h\to 0} \frac{-2a - h}{a^2(a^2+2ah+h^2)} \quad \text{Direct substitution will now work!} \\
&= \frac{-2a - 0}{a^2(a^2 + 2a(0) + (0)^2)} \\
&= \frac{-2a}{a^2(a^2)} \\
&= -\frac{2}{a^3}
\end{aligned}$$

This is the slope of the tangent line at $x = a$. The point the tangent line passes through is $(-2, 1/4)$, which means $x_0 = -2$ and $y_0 = 1/4$. That also means that our slope should be evaluated at $a = x_0 = -2$, so

$$\begin{aligned}
m &= -\left.\frac{2}{a^3}\right|_{a=-2} \\
&= -\frac{2}{(-2)^3} \\
&= \frac{1}{4}
\end{aligned}$$

The equation of the tangent line to the curve $f(x) = 1/x^2$ at the point $(-2, 1/4)$ is therefore:

$$y - y_0 = m(x - x_0) \quad \longrightarrow \quad y - \frac{1}{4} = \frac{1}{4}(x - (-2)) \quad \longrightarrow \quad y = \frac{1}{4}x + \frac{3}{4}.$$

Example Find the slope of the tangent to the curve $y = x^3 - 4x + 1$ at the point where $x = a$. Find equations of the tangent lines at the points $(1, -2)$ and $(2, 1)$. Graph the curve and both tangents.

To begin, let's talk in general before we get to the problem we are asked to solve.

The equation of the tangent line will be the equation of a straight line. The point-slope equation for a lines is

$$y - y_0 = m(x - x_0),$$

where the line passes through the point (x_0, y_0) and has slope m. The slope of the tangent line to the curve $y = f(x)$ is given by

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}.$$

We now have everything we need to solve the problem. Let's solve it!

$$\begin{aligned}
f(x) &= x^3 - 4x + 1 \\
f(a) &= a^3 - 4a + 1 \\
f(a+h) &= (a+h)^3 - 4(a+h) + 1 \\
&= a^3 + 3a^2h + 3ah^2 + h^3 - 4a - 4h + 1 \\
m &= \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \\
&= \lim_{h \to 0} \frac{(a^3 + 3a^2h + 3ah^2 + h^3 - 4a - 4h + 1) - (a^3 - 4a + 1)}{h} \\
& \qquad \text{direct substitution yields undetermined quotient} \\
&= \lim_{h \to 0} \frac{a^3 + 3a^2h + 3ah^2 + h^3 - 4a - 4h + 1 - a^3 + 4a - 1}{h} \\
&= \lim_{h \to 0} \frac{3a^2h + 3ah^2 + h^3 - 4h}{h} \\
&= \lim_{h \to 0} \frac{h(3a^2 + 3ah + h^2 - 4)}{h} \\
&= \lim_{h \to 0} (3a^2 + 3ah + h^2 - 4) \quad \text{direct substitution now works!} \\
&= 3a^2 + 3a(0) + (0)^2 - 4 \\
&= 3a^2 - 4
\end{aligned}$$

This is the slope of the tangent line at $x = a$.

The equation of the tangent line through the point $(1, -2) = (x_0, y_0)$:

$$\begin{aligned}
m &= 3a^2 - 4 \,\big|_{a=1} \\
&= 3(1)^2 - 4 \\
&= -1
\end{aligned}$$

The equation of the tangent line to the curve $f(x) = x^3 - 4x + 1$ at the point $(1, -2)$ is therefore:

$$y - y_0 = m(x - x_0) \quad \longrightarrow \quad y - (-2) = (-1)(x - (1)) \quad \longrightarrow \quad y = -x - 1.$$

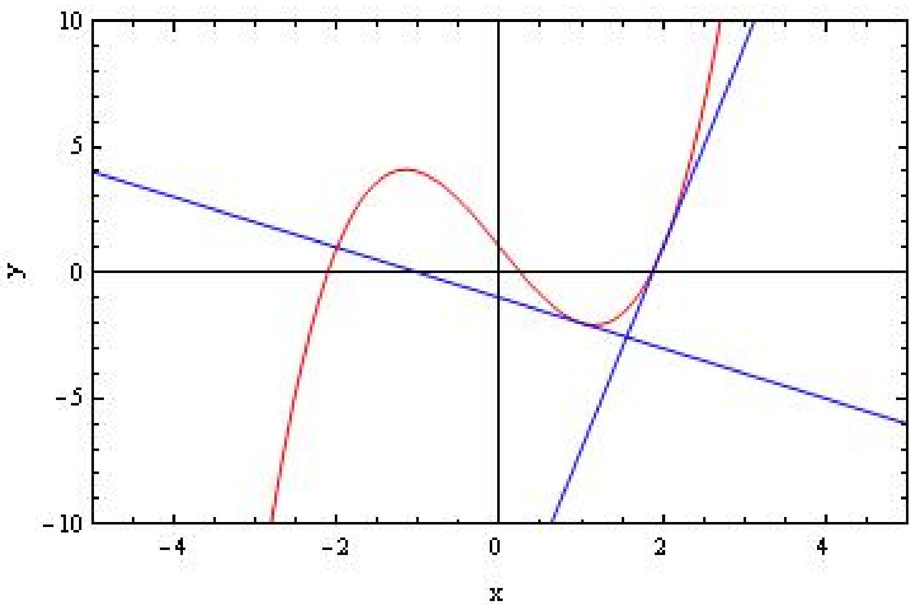
The equation of the tangent line through the point $(2, 1) = (x_0, y_0)$:

$$\begin{aligned} m &= 3a^2 - 4 \,|_{a=2} \\ &= 3(2)^2 - 4 \\ &= 8 \end{aligned}$$

The equation of the tangent line to the curve $f(x) = x^3 - 4x + 1$ at the point $(2, 1)$ is therefore:

$$y - y_0 = m(x - x_0) \quad \longrightarrow \quad y - (1) = (8)(x - (2)) \quad \longrightarrow \quad y = 8x - 15.$$

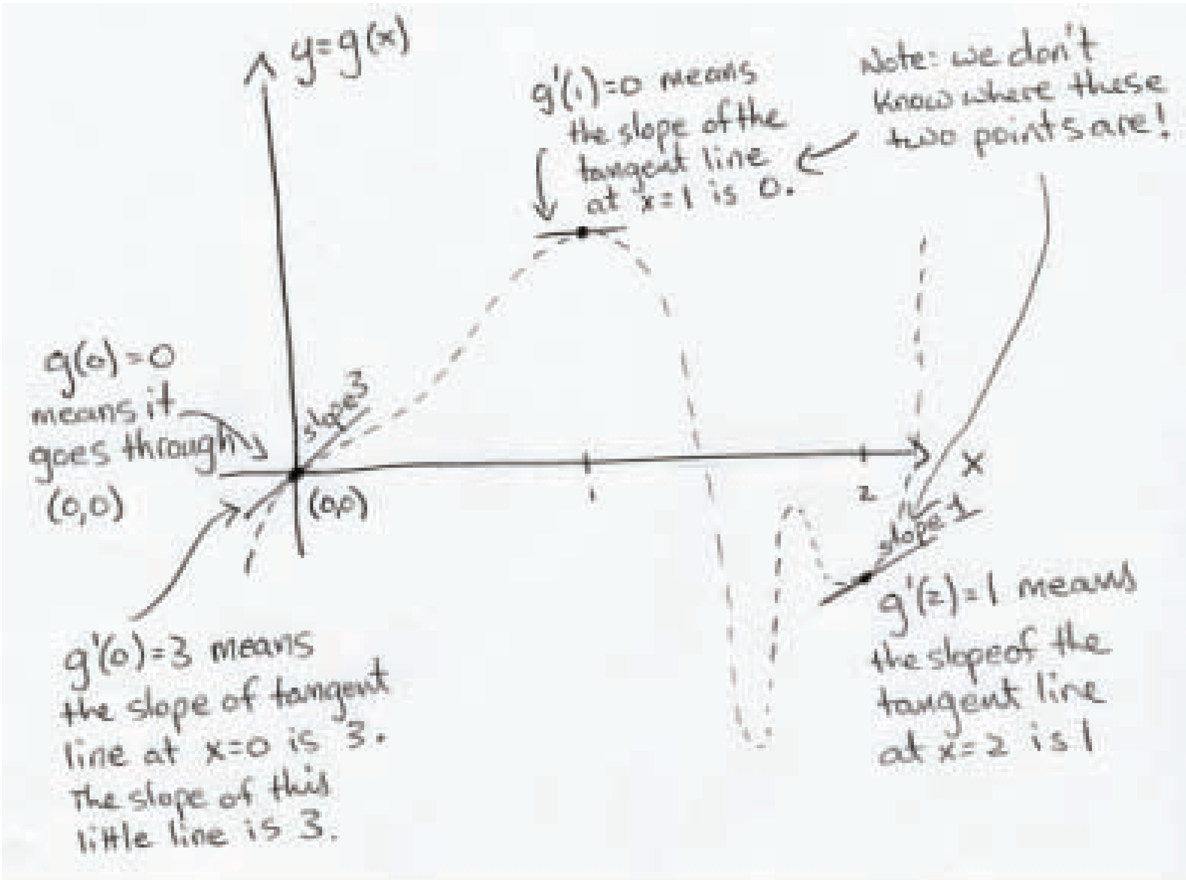
Plots and *Mathematica* commands are below.




```
plot = Plot[{x^3 - 4x + 1, -x - 1, 8x - 15}, {x, -5, 5},
    PlotStyle -> {Red,Blue,Blue},
    PlotRange -> {{-5, 5}, {-10, 10}}, Frame -> True,
    FrameLabel -> {"x", "y"}]
```

Example Sketch the graph of a function g for which $g(0) = 0$, $g'(0) = 3$, $g'(1) = 0$, and $g'(2) = 1$.

I've drawn a sketch below. Notice how I use a short tangent line to indicate the value of the derivative of the curve at that point. The slope of that tangent line is the value of derivative. Since it is a *sketch*, and not a graph with graph paper, to communicate that idea I wrote some words. What the graph is doing between the three points I've identified can be very different than what I've drawn. This is simply one curve that satisfies the properties given.



Example If $g(x) = 1 - x^3$, find $g'(0)$ and use it to find an equation of the tangent line to the curve $y = 1 - x^3$ at the point $(0, 1)$.

The definition of derivative I will use is

$$g'(a) = \lim_{h \to 0} \frac{g(a+h) - g(a)}{h}.$$

I will first find the derivative at the point $x = a$.

$$\begin{aligned}
g(a) &= 1 - a^3 \\
g(a+h) &= 1 - (a+h)^3 \\
&= 1 - (a^3 + 3a^2h + 3ah^2 + h^3) \\
&= 1 - a^3 - 3a^2h - 3ah^2 - h^3 \\
g'(a) &= \lim_{h\to 0} \frac{g(a+h) - g(a)}{h} \\
&= \lim_{h\to 0} \frac{(1 - a^3 - 3a^2h - 3ah^2 - h^3) - (1 - a^3)}{h} \\
&= \lim_{h\to 0} \frac{1 - a^3 - 3a^2h - 3ah^2 - h^3 - 1 + a^3}{h} \\
&= \lim_{h\to 0} \frac{-3a^2h - 3ah^2 - h^3}{h} \\
&= \lim_{h\to 0} \frac{h(-3a^2 - 3ah - h^2)}{h} \\
&= \lim_{h\to 0} (-3a^2 - 3ah - h^2) \quad \text{direct substitution will now work!} \\
&= -3a^2 - 3a(0) - (0)^2 = -3a^2
\end{aligned}$$

We have found that $g'(a) = -3a^2$.

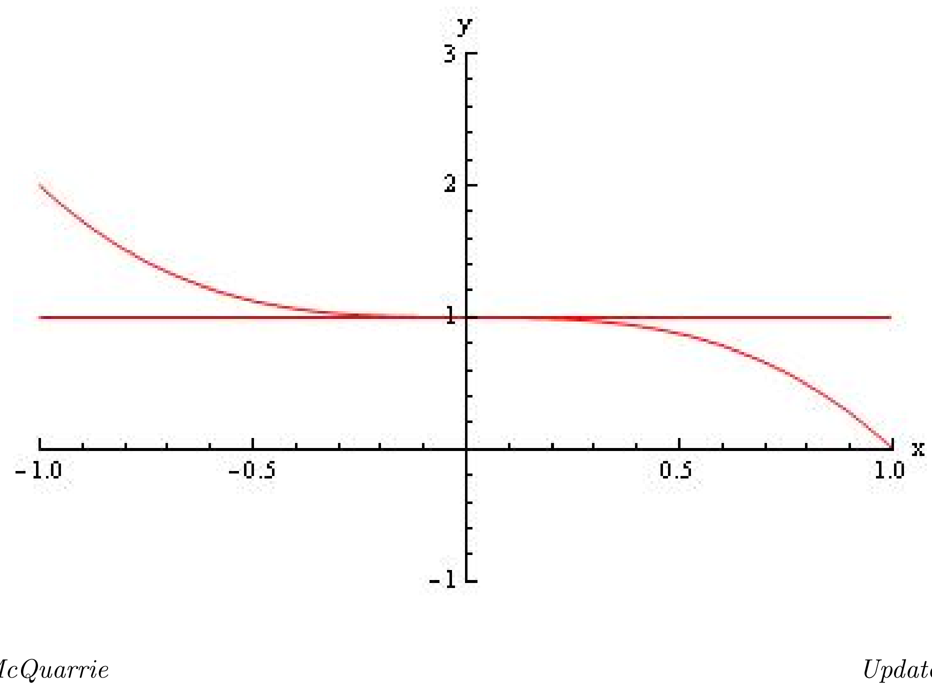
Now, we need to use this to determine the equation of the tangent line. The equation of the tangent line is given by

$$y - y_0 = m(x - x_0),$$

where (x_0, y_0) is a point on the line and m is the slope of the tangent line (which is equal to the derivative at that point). Therefore, $a = x_0 = 0$, $y_0 = 1$, and $m = g'(0) = 0$. So the equation of the tangent line is

$$y - 1 = (0)(x - 0) \quad \longrightarrow \quad y = 1.$$

The curve has a horizontal tangent at the point $(0, 1)$. I've included a sketch below.



Example Determine whether or not $f'(0)$ exists.

$$f(x) = \begin{cases} x\sin\frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

This is a tricky problem in a sense, since it contains the results from a previous problem.

If we are a bit unsure of how to start, we should always begin with some mathematical statements and see where they lead us. The key is that everything we do should be mathematically consistent.

We are asked to determine if a derivative exists. Well, let's begin by writing down the definition of derivative.

$$\begin{aligned}
f'(a) &= \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \\
f'(0) &= \lim_{h\to 0}\frac{f(h)-f(0)}{h} \\
f(0) &= 0 \\
f(h) &= h\sin(1/h) \\
f'(0) &= \lim_{h\to 0}\frac{h\sin(1/h)-0}{h} \\
&= \lim_{h\to 0}\frac{h\sin(1/h)}{h} \\
&= \lim_{h\to 0}\sin(1/h)
\end{aligned}$$

Well, that's as far as I can get algebraically. Now we need to think about how to evaluate this limit. Direct substitution will not work. In fact, the function $\sin(1/h)$ will oscillate between the values of $+1$ and -1 as h approaches zero (this is discussed in detail in Section 2.2 Example 4). So the limit does not exist.

If the limit does not exist, then the derivative does not exist.

Example Find the derivative of $f(x) = x^2 - 8x + 9$ at $x = a$.

This can be solved using either of the two forms for derivative. The first is in your text:

$$\begin{aligned}
f(a) &= a^2 - 8a + 9 \\
f(a+h) &= (a+h)^2 - 8(a+h) + 9 \\
&= a^2 + h^2 + 2ah - 8a - 8h + 9 \\
f'(a) &= \lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \\
&= \lim_{h\to 0}\frac{a^2 + h^2 + 2ah - 8a - 8h + 9 - a^2 + 8a - 9}{h} \\
&= \lim_{h\to 0}\frac{1}{h}(h^2 + 2ah - 8h) \\
&= \lim_{h\to 0}(h + 2a - 8) = 2a - 8
\end{aligned}$$

The second solution would be:

$$\begin{aligned}
f(x) &= x^2 - 8x + 9 \\
f(a) &= a^2 - 8a + 9 \\
f'(a) &= \lim_{x\to a} \frac{f(x) - f(a)}{x - a} \\
&= \lim_{x\to a} \frac{(x^2 - 8x + 9) - (a^2 - 8a + 9)}{x - a} \\
&= \lim_{x\to a} \frac{x^2 - 8x + 9 - a^2 + 8a - 9}{x - a} \\
&= \lim_{x\to a} \frac{x^2 - 8x - a^2 + 8a}{x - a} \\
&= \lim_{x\to a} \frac{x^2 - a^2 - 8(x - a)}{x - a} \\
&= \lim_{x\to a} \frac{(x + a)(x - a) - 8(x - a)}{x - a} \\
&= \lim_{x\to a} \frac{((x + a) - 8)(x - a)}{x - a} \\
&= \lim_{x\to a} ((x + a) - 8) \\
&= (a + a - 8) = 2a - 8
\end{aligned}$$

Example Find an equation of the tangent line to the parabola $y = x^2 - 8x + 9$ at the point $(5, -6)$.

Let $f(x) = y$. Then, $f'(a) = 2a - 8$ is the slope of the tangent line at $x = a$. Here, $a = 5$. $m = f'(5) = 2(5) - 8 = 2$. The point-slope equation for a line is

$$\begin{aligned}
y - y_0 &= m(x - x_0) \\
y - (-6) &= 2(x - 16) \\
y &= 2x - 16
\end{aligned}$$

is the equation of the tangent line to $f(x)$ at the point $(5, -6)$.

In *Mathematica*:

```
Plot[{x^2 - 8x + 9, 2x-16}, {x, -3, 6}]
```

Example The position of a particle is given by the equation of motion $s = f(t) = 1/(1 + t)$, where t is in seconds and s is in meters. Find the velocity and speed of the particle at $t = 2$ seconds.

I will work in general at $t = a$, and then substitute $a = 2$ at the end.

$$\begin{aligned}
f(a) &= \frac{1}{1+a} \\
f(a+h) &= \frac{1}{1+a+h} \\
f'(a) &= \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} \\
&= \lim_{h\to 0} \frac{\frac{1}{1+a+h} - \frac{1}{1+a}}{h} \\
&= \lim_{h\to 0} \frac{1}{h}\left(\frac{1}{1+a+h} - \frac{1}{1+a}\right) \\
&= \lim_{h\to 0} \frac{1}{h}\left(\frac{1+a-(1+a+h)}{(1+a+h)(1+a)}\right) \\
&= \lim_{h\to 0} \left(\frac{-1}{(1+a+h)(1+a)}\right) \\
&= \frac{-1}{(1+a)(1+a)} = \frac{-1}{(1+a)^2}
\end{aligned}$$

After 2 seconds, the velocity is therefore $f'(2) = -1/9$ m/s. The speed is the the absolute value of the velocity, so the speed is $|f'(2)| = 1/9$ m/s.

Example Find $f'(a)$ if $f(x) = \sqrt{3x+1}$.

$$\begin{aligned}
f(a) &= \sqrt{3x+1} \\
f(a+h) &= \sqrt{3(a+h)+1} \\
&= \sqrt{3a+3h+1} \\
f'(a) &= \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} \\
&= \lim_{h\to 0} \frac{\sqrt{3a+3h+1} - \sqrt{3a+1}}{h} \quad \text{Direct substitution yields indeterminant quotient} \\
&= \lim_{h\to 0} \frac{\sqrt{3a+3h+1} - \sqrt{3a+1}}{h} \cdot \left(\frac{\sqrt{3a+3h+1} + \sqrt{3a+1}}{\sqrt{3a+3h+1} + \sqrt{3a+1}}\right) \quad \text{rationalize the numerator} \\
&= \lim_{h\to 0} \frac{(3a+3h+1) - (3a+1)}{h(\sqrt{3a+3h+1} + \sqrt{3a+1})} \\
&= \lim_{h\to 0} \frac{3a+3h+1-3a-1}{h(\sqrt{3a+3h+1} + \sqrt{3a+1})} \\
&= \lim_{h\to 0} \frac{3h}{h(\sqrt{3a+3h+1} + \sqrt{3a+1})} \\
&= \lim_{h\to 0} \frac{3}{\sqrt{3a+3h+1} + \sqrt{3a+1}} \\
&= \frac{3}{\sqrt{3a+3(0)+1} + \sqrt{3a+1}} \quad \text{Direct substitution now works} \\
&= \frac{3}{2\sqrt{3a+1}}
\end{aligned}$$

Example A particle moves along a straight line with equation of motion $s = f(t) = 2t^3 - t$, where s is measured in meters and t in seconds. Find the velocity when $t = 2$.

The velocity is equal to the derivative of the position.

$$
\begin{aligned}
f'(a) &= \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} \\
f(t) &= 2t^3 - t \\
f(a) &= 2a^3 - a \\
f(a+h) &= 2(a+h)^3 - (a+h) \\
&= 2(a^3 + 3a^2h + 3ah^2 + h^3) - a - h \\
&= 2a^3 + 6a^2h + 6ah^2 + 2h^3 - a - h \\
f'(a) &= \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} \\
&= \lim_{h\to 0} \frac{(2a^3 + 6a^2h + 6ah^2 + 2h^3 - a - h) - (2a^3 - a)}{h} \quad \text{Direct substitution won't work} \\
&= \lim_{h\to 0} \frac{1}{h}[2a^3 + 6a^2h + 6ah^2 + 2h^3 - a - h - 2a^3 + a] \\
&= \lim_{h\to 0} \frac{1}{h}[6a^2h + 6ah^2 + 2h^3 - h] \\
&= \lim_{h\to 0} \frac{1}{h}[h(6a^2 + 6ah + 2h^2 - 1)] \\
&= \lim_{h\to 0} (6a^2 + 6ah + 2h^2 - 1) \quad \text{Direct substitution now works!} \\
&= 6a^2 + 6a(0) + 2(0)^2 - 1 \\
&= 6a^2 - 1
\end{aligned}
$$

The velocity when $t = 2$ s is $v(a) = f'(a) = 6a^2 - 1$. When $t = 2$, the velocity is $6(2)^2 - 1 = 23$ m/s.

Example Find an equation of the tangent line to the function $y = 5/(x - 2)$ at the point $(1, -5)$.

Let $f(x) = 5/(x-2)$. Then the slope of the tangent at $(a, f(a))$ is

$$\begin{aligned}
m &= \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \\
&= \lim_{h\to 0} \frac{\frac{5}{(a+h)-2} - \frac{5}{a-2}}{h} = \lim_{h\to 0} \frac{\left(\frac{5}{a+h-2} - \frac{5}{a-2}\right)}{h} \\
&= \lim_{h\to 0} \frac{1}{h}\left(\left(\frac{5}{a+h-2}\right)\cdot\left(\frac{a-2}{a-2}\right) - \left(\frac{5}{a-2}\right)\cdot\left(\frac{a+h-2}{a+h-2}\right)\right) \\
&= \lim_{h\to 0} \frac{1}{h}\left(\frac{5(a-2)-5(a+h-2)}{(a-2)(a+h-2)}\right) \\
&= \lim_{h\to 0} \frac{1}{h}\left(\frac{5a-10-5a-5h+10)}{(a-2)(a+h-2)}\right) \\
&= \lim_{h\to 0} \frac{1}{h}\left(\frac{-5h}{(a-2)(a+h-2)}\right) \\
&= -\lim_{h\to 0} \frac{h}{h}\left(\frac{5}{(a-2)(a+h-2)}\right) \\
&= -\lim_{h\to 0} \left(\frac{5}{(a-2)(a+h-2)}\right) \\
&= -\left(\frac{5}{(a-2)(a+0-2)}\right) \\
&= -\frac{5}{(a-2)^2}
\end{aligned}$$

We are interested in $a = 1$, so the slope is $m = -5$.

Use the point slope form of the equation of a line: $y - y_1 = m(x - x_1)$.
Therefore, the equation of the tangent line at the point (1,-5) is $y - (-5) = -5(x-1) \longrightarrow y = -5x$.

Example The position of a particle is given by $s(t) = \sqrt{t^2+1}$ where s is measured in meters and t is measured in seconds. What is the instantaneous velocity of the particle when $t = 1$ second?

The instantaneous velocity when $t = a$ seconds is given by:

$$\begin{aligned}
v(a) &= \lim_{h\to 0}\frac{s(a+h)-s(a)}{h} \\
&= \lim_{h\to 0}\frac{\sqrt{(a+h)^2+1}-\sqrt{a^2+1}}{h} \\
&= \lim_{h\to 0}\frac{\sqrt{a^2+h^2+2ah+1}-\sqrt{a^2+1}}{h} \\
&= \lim_{h\to 0}\frac{\sqrt{a^2+h^2+2ah+1}-\sqrt{a^2+1}}{h}\cdot\frac{\sqrt{a^2+h^2+2ah+1}+\sqrt{a^2+1}}{\sqrt{a^2+h^2+2ah+1}+\sqrt{a^2+1}} \\
&= \lim_{h\to 0}\frac{(a^2+h^2+2ah+1)-(a^2+1)}{h(\sqrt{a^2+h^2+2ah+1}+\sqrt{a^2+1})} \\
&= \lim_{h\to 0}\frac{h^2+2ah}{h(\sqrt{a^2+h^2+2ah+1}+\sqrt{a^2+1})} \\
&= \lim_{h\to 0}\frac{h(h+2a)}{h(\sqrt{a^2+h^2+2ah+1}+\sqrt{a^2+1})} \\
&= \lim_{h\to 0}\frac{(h+2a)}{(\sqrt{a^2+h^2+2ah+1}+\sqrt{a^2+1})} \\
&= \frac{(0+2a)}{(\sqrt{a^2+0^2+2a(0)+1}+\sqrt{a^2+1})} \\
&= \frac{2a}{2\sqrt{a^2+1}} \\
&= \frac{a}{\sqrt{a^2+1}}
\end{aligned}$$

At $a = 1$ second, $v(1) = \dfrac{1}{\sqrt{2}}$ m/s. The units come from the definition.