Questions

Example Differentiate $y = \dfrac{\tan x - 1}{\sec x}$.

Example Prove that $\dfrac{d}{dx}(\sec x) = \sec x \tan x$.

Example If $f(x) = 2x + \cot x$, find $f'(x)$. Check to see if your answer in part (a) is reasonable by graphing both f and f' for $0 < x < \pi$.

Solutions

Example Differentiate $y = \dfrac{\tan x - 1}{\sec x}$.

$$
\begin{aligned}
y &= \frac{\tan x - 1}{\sec x} \\
\frac{dy}{dx} &= \frac{d}{dx}\left[\frac{\tan x - 1}{\sec x}\right] \\
&= \frac{\sec x \frac{d}{dx}(\tan x - 1) - (\tan x - 1)\frac{d}{dx}(\sec x)}{\sec^2 x} \\
&= \frac{\sec x(\sec^2 x) - (\tan x - 1)(\sec x \tan x)}{\sec^2 x} \\
&= \frac{\sec^3 x - (\sec x \tan^2 x - \sec x \tan x)}{\sec^2 x} \\
&= \frac{\sec^3 x - \sec x \tan^2 x + \sec x \tan x}{\sec^2 x} \\
&= \frac{\sec^2 x - \tan^2 x + \tan x}{\sec x} \\
&= \frac{1 + \tan x}{\sec x} \text{ (since } \sec^2 x - \tan^2 x = 1) \\
&= \cos x(1 + \tan x) \\
&= \cos x\left(1 + \frac{\sin x}{\cos x}\right) \\
&= \cos x + \sin x
\end{aligned}
$$

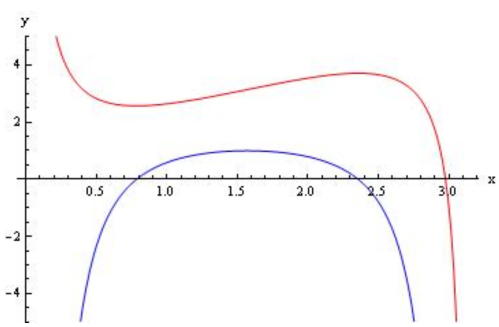
Example Prove that $\dfrac{d}{dx}(\sec x) = \sec x \tan x$.

$$\begin{aligned}
\frac{d}{dx}(\sec x) &= \frac{d}{dx}\left(\frac{1}{\cos x}\right) \\
&= \frac{\cos x \frac{d}{dx}[1] - (1)\frac{d}{dx}[\cos x]}{\cos^2 x} \\
&= \frac{\cos x(0) - (1)(-\sin x)}{\cos^2 x} \\
&= \frac{\sin x}{\cos^2 x} \\
&= \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} \\
&= \sec x \tan x
\end{aligned}$$

Example If $f(x) = 2x + \cot x$, find $f'(x)$. Check to see if your answer in part (a) is reasonable by graphing both f and f' for $0 < x < \pi$.

$$\begin{aligned}
f(x) &= 2x + \cot x \\
f'(x) &= \frac{d}{dx}[2x + \cot x] \\
&= 2 - \csc^2 x
\end{aligned}$$

Here is a plot of the function (in red) and its derivative (in blue).



The *Mathematica* commands I used to create this plot were

```
Plot[{2x + Cot[x], 2 - Csc[x]^2}, {x, 0, Pi},
  PlotRange -> {Automatic, {-5, 5}},
  PlotStyle -> {Red,Blue},
  AxesLabel -> {"x", "y"}]
```

As always, other than the `PlotRange`, you would not need to use the extra options I used to format my plot.

We can see that the the derivative is zero where the function has horizontal tangents, and the derivative is positive where the function is increasing and negative where the function is decreasing. Our derivative seems reasonable based on the graphs.