**Questions**

**Example** Find $\lim_{x\to\infty} \frac{e^x}{x}$.

**Example** Find $\lim_{x\to\infty} x\tan(1/x)$.

**Example** Find $\lim_{x\to 1}\left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$.

**Solutions**

**Example** Find $\lim_{x\to\infty} \frac{e^x}{x}$.

$$\begin{aligned}
\lim_{x\to\infty} \frac{e^x}{x} &\to \frac{\infty}{\infty} \quad \text{(direct substitution leads to indeterminant quotient; use l'H.R.)}\\
\lim_{x\to\infty} \frac{e^x}{x} &= \lim_{x\to\infty} \frac{\frac{d}{dx}[e^x]}{\frac{d}{dx}[x]}\\
&= \lim_{x\to\infty} \frac{e^x}{1}\\
&= \lim_{x\to\infty} e^x \to \infty\\
\lim_{x\to\infty} \frac{e^x}{x} &= \infty
\end{aligned}$$

**Example** Find $\lim_{x\to\infty} x\tan(1/x)$.

$$\begin{aligned}
\lim_{x\to\infty} x\tan(1/x) &\to \infty\cdot\tan 0 \to \infty\cdot 0 \quad \text{(indeterminant product; cannot use l'H.R.)}\\
\lim_{x\to\infty} x\tan(1/x) &= \lim_{x\to\infty} \frac{\tan(1/x)}{1/x} \to \frac{0}{0} \quad \text{(indeterminant quotient; can now use l'H.R.)}\\
&= \lim_{x\to\infty} \frac{\frac{d}{dx}[\tan(1/x)]}{\frac{d}{dx}[1/x]}\\
&= \lim_{x\to\infty} \frac{\sec^2(1/x)(-1/x^2)}{(-1/x^2)}\\
&= \lim_{x\to\infty} \sec^2(1/x)\\
&= \sec^2(0) = \frac{1}{\cos^2 0} = 1\\
\lim_{x\to\infty} x\tan(1/x) &= 1
\end{aligned}$$

**Example** Find $\lim_{x\to 1}\left(\frac{1}{\ln x} - \frac{1}{x-1}\right)$.

$$\lim_{x\to 1}\left(\frac{1}{\ln x} - \frac{1}{x-1}\right) \to \infty - \infty \quad \text{(indeterminant difference; cannot use l'H.R.)}$$

$$\begin{aligned}
\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right) &= \lim_{x\to 1}\left(\frac{x-1-\ln x}{(x-1)\ln x}\right)\to\frac{0}{0} \quad \text{(indeterminant quotient; can now use l'H.R.)}\\
&= \lim_{x\to 1}\left(\frac{\frac{d}{dx}[x-1-\ln x]}{\frac{d}{dx}[(x-1)\ln x]}\right)\\
&= \lim_{x\to 1}\left(\frac{1-1/x}{(x-1)\frac{d}{dx}[\ln x]+\ln x\frac{d}{dx}[x-1]}\right)\\
&= \lim_{x\to 1}\left(\frac{1-1/x}{(x-1)(1/x)+\ln x(1)}\right)\\
&= \lim_{x\to 1}\left(\frac{1-1/x}{1-1/x+\ln x}\right)\to\frac{0}{0} \quad \text{(indeterminant quotient; can use l'H.R.)}\\
&= \lim_{x\to 1}\left(\frac{\frac{d}{dx}[1-1/x]}{\frac{d}{dx}[1-1/x+\ln x]}\right)\\
&= \lim_{x\to 1}\left(\frac{1/x^2}{1/x^2+1/x}\right)\\
&= \frac{1}{1+1}=\frac{1}{2}\\
\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right) &= \frac{1}{2}
\end{aligned}$$