Questions

Example Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function

$$y = \int_{1-3x}^{1} \frac{u^3}{1+u^2}\, du.$$

Example Use Part 2 of the Fundamental Theorem of Calculus to evaluate the integral, or explain why it does not exist,

$$\int_0^4 (1 + 3y - y^2)\, dy.$$

Example Find the interval on which the curve $y = \displaystyle\int_0^x \frac{1}{1+t+t^2}\, dt$ is concave up.

Solutions

Example Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the function

$$y = \int_{1-3x}^{1} \frac{u^3}{1+u^2}\, du.$$

This will require the chain rule.

Let $g(v) = \displaystyle\int_v^1 \frac{u^3}{1+u^2}\, du = \int_1^v \frac{-u^3}{1+u^2}\, du$. Then $g'(v) = \dfrac{dg}{dv} = \dfrac{-v^3}{1+v^2}$ by the FTC Part 1.

However, we have $y = g(1-3x)$.
Therefore, $\dfrac{dy}{dx} = \dfrac{d}{dx} g(1-3x) = g'(1-3x)\dfrac{d}{dx}(1-3x) = \dfrac{-(1-3x)^3}{1+(1-3x)^2}(-3) = \dfrac{3(1-3x)^3}{1+(1-3x)^2}$.

Example Use Part 2 of the Fundamental Theorem of Calculus to evaluate the integral, or explain why it does not exist,

$$\int_0^4 (1 + 3y - y^2)\, dy.$$

$$\begin{aligned}
\int_0^4 (1 + 3y - y^2)\, dy &= \left(y + \frac{3}{2}y^2 - \frac{1}{3}y^3\right)_0^4 \\
&= \left((4) + \frac{3}{2}(4)^2 - \frac{1}{3}(4)^3\right) - \left((0) + \frac{3}{2}(0)^2 - \frac{1}{3}(0)^3\right) \\
&= \left(4 + \frac{48}{2} - \frac{64}{3}\right) \\
&= \frac{20}{3}
\end{aligned}$$

Example Find the interval on which the curve $y = \int_0^x \frac{1}{1+t+t^2}\,dt$ is concave up.

This curve defines y as a function of x. For a curve to be concave up, we must have $y'' > 0$.

$$\begin{aligned}
y &= \int_0^x \frac{1}{1+t+t^2}\,dt \\
y' &= \frac{1}{1+x+x^2} \\
y'' &= \frac{(1+x+x^2)\frac{d}{dx}[1] - (1)\frac{d}{dx}[1+x+x^2]}{(1+x+x^2)^2} \\
&= \frac{-(1+2x)}{(1+x+x^2)^2}
\end{aligned}$$

So $y'' > 0$ if $-(1+2x) > 0$, or $x < -1/2$.

Here is a sketch:

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y := Integrate[1/(1 + t + t^2), {t, 0, x}]
Plot[y, {x, -2, 2}]
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