You should be *expanding* this study guide as you see fit with details and worked examples. With this extra layer of detail you will then have excellent study notes for exams, and later reference.

Note that we will not be covering these sections in strictly numerical order, and some topics will be emphasized more than others.

Practice is suggested from Stewart *Calculus, early transcendentals*, 6th Edition.

Mathematica

- The library of built-in *Mathematica* commands we understand and use will grow throughout the semester. Keep a list of the basic commands we use and practice using *Mathematica*! Use the *Mathematica Quick Reference* which was handed out in class, and use *Mathematica* to check homework whenever possible, rather than wolframalpha.

1.1 Four Ways to Represent a Function (WEEK OF JAN 13)

- relations and functions (vertical line test)
- functional notation $y = f(x)$ (x continuous)
- domain for $y = f(x)$ is set D (x values) and range is set R (y values)
- average rate of change (difference quotient) on interval $[a, a+h]$ is $\frac{\Delta y}{\Delta x} = \frac{f(a+h)-f(a)}{h}$
- piecewise defined functions such as $f(x) = |x|$
- graphs of functions $y = f(x)$
- even: $f(-x) = f(x)$ for all $x \in D$; odd: $f(-x) = -f(x)$ for all $x \in D$; neither.
- increasing and decreasing (on an interval)
- Practice: 1.1.30, 1.1.41, 1.1.57, 1.1.65.

1.2 Mathematical Models: Catalog of Essential Functions (WEEK OF JAN 13)

- polynomials $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$
- quadratics $f(x) = x^2$, $f(x) = ax^2 + bx + c$ and quadratic formula
- cubics $f(x) = x^3$ and $f(x) = ax^3 + bx^2 + cx + d$
- power function $f(x) = x^n$, $n \in \mathbb{N}_1$ (the natural numbers starting at 1 (positive integers))
- reciprocal function $f(x) = x^{1/n}$, $n \in \mathbb{N}_1$; $f(x) = \sqrt{x}$ and $f(x) = \sqrt[3]{x} = x^{1/3}$
- rational functions $f(x) = \frac{P(x)}{Q(x)}$, where P and Q are polynomials
- algebraic functions
- trig functions (important when we get to section 3.3)
- exponential functions $f(x) = e^x$
- logarithmic functions $f(x) = \ln(x)$
- transcendental functions
- Practice: 1.2.3, 1.2.8, 1.2.15

1.3 New Functions from Old Functions (WEEK OF JAN 20)

- transformations (graphical and algebraic) of $y = f(x)$:
  horizontal shift to right: $y = f(x-h)$, $h > 0$
  horizontal shift to left: $y = f(x+h)$, $h > 0$
  vertical shift up: $y = f(x) + k$, $k > 0$
  vertical shift down: $y = f(x) - k$, $k > 0$
  reflect about x axis: $y = -f(x)$
  reflect about y axis: $y = f(-x)$
  stretch vertically: $y = cf(x)$, $c > 1$
  compress horizontally: $y = f(cx)$, $c > 1$
- notice: anything outside of f is vertical, anything inside of f is horizontal and opposite

- multiple translations, for example $y = a(x-h)^2 + k$
- new functions from old:
  sum $(f+g)(x) = f(x) + g(x)$
  difference $(f-g)(x) = f(x) - g(x)$
  product $(fg)(x) = f(x)g(x)$
  quotient $(f/g)(x) = f(x)/g(x)$
  composition $(f \circ g)(x) = f(g(x))$
- determine domain by looking at unsimplified form and checking for:
  division by zero, square root of negative, logarithm of negative
- determining the domain often requires solving compound inequalities
- note $f \circ g \neq g \circ f$ (not commutative)
- Practice: 1.3.3, 1.3.5, 1.3.35, 1.3.39, 1.3.48, 1.3.63

1.4 Graphing Calculators and Computers (WEEK OF JAN 20)

- *Mathematica* Quick Reference on moodle:
  Arithmetic, Brackets, Built-In Functions, Built-In Constants & Symbols, Equal Sign, Solving Equations, Defining Your Own Function
- basic *Mathematica* syntax on moodle
- Practice: on moodle

1.5 Exponential Functions (WEEK OF JAN 20)

- the number $e \sim 2.71828$
- rules of exponents:
  $a^0 = 1$ if $a \neq 0$
  $a^m a^n = a^{m+n}$ (product same base)
  $\frac{a^m}{a^n} = a^{m-n}$ (quotient same base)
  $(a^m)^n = a^{mn}$ (power)
  $(ab)^n = a^n b^n$ (product different base)
  $(\frac{a}{b})^n = \frac{a^n}{b^n}$ (quotient different base)
  $a^{1/n} = \sqrt[n]{a}$ (fractional exponent)
  $\sqrt[n]{ab} = \sqrt[n]{a}\sqrt[n]{b}$ if $a, b \in \mathbb{R}$ and n is positive integer (radical notation)
- Practice: 1.5.15, 1.5.19, 1.5.20, 1.5.29

1.6 Inverse Functions and Logarithms (WEEK OF JAN 27)

- definition of one-to-one functions (horizontal line test)
- inverse functions (both graphically, and algebraically)
- If $f(x)$ has domain D and range R, then $f^{-1}(x)$ has domain R and range D
- notation for inverse function: $f^{-1}(x) \neq \frac{1}{f(x)}$
- check using $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$
- rules of logarithms:
  $\ln e^x = x$ and $e^{\ln x} = x$ (inverse)
  $\ln(MN) = \ln(M) + \ln(N)$ (product)
  $\ln(\frac{M}{N}) = \ln(M) - \ln(N)$ (quotient)
  $\ln(M^N) = N\ln(M)$ (power)
- change of base
- inverse trigonometric functions $\arcsin(x)$, $\arccos(x)$, $\arctan(x)$ (important in Section 3.5)
- Practice: 1.6.18, 1.6.20, 1.6.49, 1.6.53, 1.6.55

2.1 Tangent and Velocity Problems (WEEK OF JAN 27)

- tangent problem: definition of tangent line to a curve $y = f(x)$ at a point $P(a, f(a))$
- velocity problem: definition of instantaneous velocity
- Practice: 2.1.3, 2.1.5

2.2 The Limit of a Function (WEEK OF FEB 3)

- definition of limit $\lim\limits_{x\to a} f(x) = L$ (calculators may give false values)
- definition of left-hand limit $\lim\limits_{x\to a^-} f(x) = L$ and right-hand limit $\lim\limits_{x\to a^+} f(x) = L$
- definition of infinite limits $\lim\limits_{x\to a} f(x) = \infty$ and $\lim\limits_{x\to a} f(x) = -\infty$
- vertical asymptotes
- *Mathematica* `Limit`
- Practice: 2.2.4, 2.2.6, 2.2.7, 2.2.12, 2.2.26, 2.2.40

2.3 Calculating Limits Using Limit Laws (WEEK OF FEB 3)

- If c is a constant and $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist, then:
  $\lim\limits_{x\to a}[f(x) + g(x)] = \lim\limits_{x\to a} f(x) + \lim\limits_{x\to a} g(x)$
  $\lim\limits_{x\to a}[f(x) - g(x)] = \lim\limits_{x\to a} f(x) - \lim\limits_{x\to a} g(x)$
  $\lim\limits_{x\to a}[cf(x)] = c \lim\limits_{x\to a} f(x)$
  $\lim\limits_{x\to a}[f(x) \cdot g(x)] = \lim\limits_{x\to a} f(x) \cdot \lim\limits_{x\to a} g(x)$
  $\lim\limits_{x\to a}[f(x)/g(x)] = \lim\limits_{x\to a} f(x)/ \lim\limits_{x\to a} g(x)$ if $\lim\limits_{x\to a} g(x) \neq 0$
  $\lim\limits_{x\to a}[f(x)]^n = \left[\lim\limits_{x\to a} f(x)\right]^n$
  $\lim\limits_{x\to a}[c] = c$ $\quad\lim\limits_{x\to a}[x] = a$ $\quad\lim\limits_{x\to a}[x^n] = a^n$
  $\lim\limits_{x\to a}[\sqrt[n]{x}] = \sqrt[n]{a}$ where n is positive integer, assume $a > 0$ if n is even
  $\lim\limits_{x\to a}[\sqrt[n]{f(x)}] = \sqrt[n]{f(a)}$ where n is positive integer, assume $\lim\limits_{x\to a} f(x) > 0$ if n is even
- direct substitution: if f is a polynomial or rational function and a is in domain of f, $\lim\limits_{x\to a} f(x) = f(a)$
- THEOREM: $\lim\limits_{x\to a} f(x) = L$ if and only if $\lim\limits_{x\to a^-} f(x) = \lim\limits_{x\to a^+} f(x) = L$
- THEOREM: If $f(x) \leq g(x)$ when x is near a (except possibly at a) and the limits exist, then $\lim\limits_{x\to a} f(x) \leq \lim\limits_{x\to a} g(x)$
- SQUEEZE THEOREM: If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a) and $\lim\limits_{x\to a} f(x) = \lim\limits_{x\to a} h(x) = L$ then $\lim\limits_{x\to a} g(x) = L$
- Practice: 2.3.11, 2.3.17, 2.3.25, 2.3.31, 2.3.44, 2.3.47, 2.3.52, 2.3.61

2.5 Continuity (WEEK OF FEB 3)

- Continuity: if $\lim\limits_{x\to a} f(x) = f(a)$ then f is continuous at $x = a$
- definition of continuous from left; continuous from right; continuous on an interval
- removable, infinite, and jump discontinuities
- polynomials are continuous for $x \in \mathbb{R}$
- rational, root, trig, inverse trig, exponential, and logarithmic functions are continuous for x in their domain
- THEOREM: if f is continuous at b and $\lim\limits_{x\to a} g(x) = b$ then $\lim\limits_{x\to a} f(g(x)) = f\left(\lim\limits_{x\to a} g(x)\right)$
- THEOREM: if g is continuous at a and f is continuous at $g(a)$, then $f(g(x))$ is continuous at a
- THE INTERMEDIATE VALUE THEOREM: Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number $c \in (a, b)$ such that $f(c) = N$.
- Practice: 2.5.3, 2.5.29, 2.5.40, 2.5.42, 2.5.61

2.6 Limits at Infinity; Horizontal Asymptotes (WEEK OF FEB 3)

- limits at infinity $\lim_{x\to\infty} f(x) = L$ and $\lim_{x\to-\infty} f(x) = L$ (horizontal asymptotes)
- if $r > 0$ is rational, $\lim_{x\to\infty} \frac{1}{x^r} = 0$
- if $r > 0$ is rational such that x^r is defined for all x, $\lim_{x\to-\infty} \frac{1}{x^r} = 0$
- infinite limits at infinity $\lim_{x\to\infty} f(x) = \infty$, $\lim_{x\to\infty} f(x) = -\infty$, $\lim_{x\to-\infty} f(x) = \infty$ and $\lim_{x\to-\infty} f(x) = -\infty$
- Practice: 2.6.3, 2.6.9, 2.6.17, 2.6.23, 2.6.29, 2.6.31, 2.6.33, 2.6.37, 2.6.56

2.7 Derivatives and Rates of Change (WEEK OF FEB 10)

- slope of secant line through points $P(a, f(a))$ and $Q(x, f(x))$: $m_{PQ} = \frac{f(x) - f(a)}{x - a}$
- slope of tangent line through point $P(a, f(a))$: $m = \lim_{x\to a} \frac{f(x) - f(a)}{x - a} = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$
- average velocity on interval $t \in [a, a+h]$ where $f(t)$ is a position function: $v_{\text{avg}} = \frac{f(a+h) - f(x)}{h}$
- instantaneous velocity at $t = a$: $v(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$
- average rate of change of f on interval $[x_1, x_2]$ is $\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$
- instantaneous rate of change of f at x_1 is $\lim_{\Delta x\to 0} \frac{\Delta y}{\Delta x} = \lim_{x_2\to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$
- the derivative of f at a is defined as $f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} = \lim_{x\to a} \frac{f(x) - f(a)}{x - a}$
- Practice: 2.7.9, 2.7.25, 2.7.29, 2.7.33, 2.7.51

2.8 The Derivative as a Function (WEEK OF FEB 10)

- the derivative of f is $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$
- other notations: Leibniz $f'(x) = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}[f(x)]$; $f'(a) = \left.\frac{dy}{dx}\right|_{x=a}$
- other notations: $f'(x) = y' = D[f(x)] = D_x[f(x)]$
- THEOREM: If f is differentiable at a, then f is continuous at a (converse of this theorem is false)
- how a function can fail to be differentiable (sharp corner, discontinuity, vertical tangent)
- higher derivatives $y^{(n)} = f^{(n)}(x) = \frac{d^n y}{dx^n}$
- position $y = f(x)$, velocity $\frac{dy}{dx} = f'(x)$, acceleration $\frac{d^2y}{dx^2} = f''(x)$, jerk $\frac{d^3y}{dx^3} = f'''(x)$
- *Mathematica* `D[f[x],x]` and `f'[x]`
- Practice: 2.8.9, 2.8.24, 2.8.26, 2.8.39, 2.8.43

3.1 Derivatives of Polynomial and Exponential Functions (WEEK OF FEB 10)

- Derivative Rules (memorize and prove)
  $\frac{d}{dx}[c] = 0$ (constant function)
  $\frac{d}{dx}[x^n] = nx^{n-1}$ (power rule)
  $\frac{d}{dx}[cf(x)] = c\frac{d}{dx}[f(x)]$ (constant multiple rule)
  $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$ (sum rule and difference rule)
  $\frac{d}{dx}[e^x] = e^x$ (natural exponential function; definition of the number $e \sim 2.71828$)
- Practice: 3.1.15, 3.1.28, 3.1.35, 3.1.39, 3.1.53, 3.1.65

3.2 The Product and Quotient Rules (WEEK OF FEB 17)

- Derivative Rules (memorize and prove)

  $\frac{d}{dx}[f(x) \cdot g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$ (product rule)

  $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$ (quotient rule: lo-de-hi minus hi-de-lo over lo squared)
- Practice: 3.2.7, 3.2.17, 3.2.24, 3.2.55, 3.2.58

3.3 Derivatives of Trig Functions (WEEK OF FEB 17)

- Derivative Rules (memorize and prove)

  $\frac{d}{dx}[\sin x] = \cos x$

  $\frac{d}{dx}[\cos x] = -\sin x$

  $\frac{d}{dx}[\tan x] = \sec^2 x$

  $\frac{d}{dx}[\sec x] = \sec x \tan x$

  $\frac{d}{dx}[\csc x] = -\csc x \cot x$

  $\frac{d}{dx}[\cot x] = -\csc^2 x$
- Practice: 3.3.12, 3.3.16, 3.3.26, 3.3.49

3.4 The Chain Rule (WEEK OF FEB 17)

- Derivative Rules (memorize and prove)

  if $y = f(u)$ and $u = g(x)$, then $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ (chain rule)

  $\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}\frac{d}{dx}[g(x)]$ (power rule with chain rule)

  $\frac{d}{dx}[a^x] = a^x \ln(a)$ (exponential with base $a > 0$ and chain rule)
- Practice: 3.4.1, 3.4.6, 3.4.15, 3.4.20, 3.4.34, 3.4.52, 3.4.84

3.5 Implicit Differentiation (WEEK OF FEB 17)

- implicit differentiation of $F(x, y) = G(x, y)$ to determine $\frac{dy}{dx}$ is performed by $\frac{d}{dx}[F(x, y)] = \frac{d}{dx}[G(x, y)]$ where you use chain rule whenever you encounter $y = y(x)$
- Derivative rules (memorize and prove)

  $\frac{d}{dx}[\arcsin(x)] = \frac{1}{\sqrt{1-x^2}}$

  $\frac{d}{dx}[\arccos(x)] = -\frac{1}{\sqrt{1-x^2}}$

  $\frac{d}{dx}[\arctan(x)] = \frac{1}{1+x^2}$

  $\frac{d}{dx}[\sec^{-1}(x)] = \frac{1}{x\sqrt{x^2-1}}$

  $\frac{d}{dx}[\csc^{-1}(x)] = -\frac{1}{x\sqrt{x^2-1}}$

  $\frac{d}{dx}[\cot^{-1}(x)] = -\frac{1}{1+x^2}$
- *Mathematica* `D[x*y[x] == Cos[x*y], x]` and then do `Solve[%, y'[x]]`
- Practice: 3.5.10, 3.5.14, 3.5.25, 3.5.26, 3.5.32, 3.5.38, 3.5.40, 3.5.51, 3.5.57, 3.5.64

3.6 Derivatives of Logarithmic Functions (WEEK OF FEB 24)

- logarithmic differentiation process:
  take logarithms of both sides of equation and simplify; implicitly differentiate; solve for y'
- use logarithmic differentiation to prove the power rule of derivatives, determine dy/dx when x is in both base and power, simplify complex product/quotient rule derivatives
- Derivative rules (memorize and prove)
  $\frac{d}{dx}[\log_a(x)] = \frac{1}{x\ln(a)}$
  $\frac{d}{dx}[\ln|x|] = \frac{1}{x}$ (note the absolute value)
- the number e as a limit $e = \lim_{x\to 0}(1+x)^{1/x} = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$
- Practice: 3.6.13, 3.6.39, 3.6.47, 3.6.50, 3.6.53

3.7 Rates of Change in Natural and Social Sciences (WEEK OF FEB 24)

- physics, chemistry, biology, economics
- Practice: 3.7.12, 3.7.13, 3.7.16, 3.7.23, 3.7.33

3.9 Related Rates (WEEK OF FEB 24)

- Think of these as "Related Rates of Change" word problems, which are solved by the following process:
  1. Draw a diagram (this usually helps) and introduce notation for all quantities.
  2. Express any given information and the required rate of change in terms of derivatives using your notation.
  3. From your diagram, determine an equation relating the quantities in the problem. This equation will not have derivatives in it.
  4. Eliminate variables if necessary.
  5. Differentiate (usually implicitly, and use chain rule as needed) to obtain an equation relating rates of change. Do not substitute given information until after you have differentiated.
  6. Substitute the given information and solve for the unknown rate of change.
- Practice: 3.9.15, 3.9.17, 3.9.23, 3.9.33, 3.9.43

3.10 Linear Approximation and Differentials (WEEK OF MAR 3)

- linear (or tangent line) approximation to f at a is $y = f(a) + f'(a)(x-a)$
- $\sin(x) \sim x$ if $x \sim 0$
- differentials: if $y = f(x)$ then $dy = f'(x)dx$ where dx is any real number
- Diagram relating dx and dy to Δx and Δy
- Practice: 3.10.2, 3.10.8, 3.10.10, 3.10.37, 3.10.41

4.1 Maximum and Minimum Values (WEEK OF MAR 3)

- absolute (or global) maximum and minimum
- local (or relative) maximum and minimum
- EXTREME VALUE THEOREM: If f is continuous on $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ for some c and d in $[a, b]$.
- FERMAT'S THEOREM: If f has a local max or min at c, and if $f'(c)$ exists, then $f'(c) = 0$.
- a critical number of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.
- The Closed Interval Method
- Practice: 4.1.21, 4.1.35, 4.1.57, 4.1.61, 4.1.73

4.2 The Mean Value Theorem (week of mar 17)

- Rolle's Theorem: Let f be a function that is continuous on $[a,b]$ and differentiable on (a,b), and $f(a)=f(b)$. Then there exists a number $c \in (a,b)$ such that $f'(c)=0$.
- The Mean Value Theorem: Let f be a function that is continuous on $[a,b]$ and differentiable on (a,b). Then there exists a number $c \in (a,b)$ such that $f'(c)=\frac{f(b)-f(a)}{b-a}$.
- Practice: 4.2.5, 4.2.14, 4.2.33, 4.2.35, 4.2.36

4.3 How Derivatives Affect Shape of Graph (week of mar 17)

- sign charts or interval tables
- f is concave up on an interval if it lies above all its tangents in the interval; concave down if it lies below all its tangent lines
- f has an inflection point at $P(c,f(c))$ if f is continuous at c and the concavity changes at c
- Increasing/Decreasing Test:
  If $f'(x)>0$ on an interval, then f is increasing on the interval.
  If $f'(x)<0$ on an interval, then f is decreasing on the interval.
- The First Derivative Test: Suppose c is a critical number of f.
  If f' changes from positive to negative at c, then f has local max at c.
  If f' changes from negative to positive at c, then f has local min at c.
  If f' does not change sign at c, then f has no local extrema at c.
- Concavity Test:
  If $f''^{(x)}>0$ on an interval, then f is concave up on the interval.
  If $f''^{(x)}<0$ on an interval, then f is concave down on the interval.
- The Second Derivative Test: Suppose f'' is continuous near c.
  If $f'(c)=0$ and $f''^{(c)}>0$, then f has local min at c.
  If $f'(c)=0$ and $f''^{(c)}<0$, then f has local max at c.
- Practice: 4.3.33, 4.3.37, 4.3.49, 4.3.65

4.4 Indeterminant Forms and l'Hospital's Rule (week of mar 17)

- Types of indeterminant forms: quotient: $\frac{0}{0}, \frac{\infty}{\infty}$ product: $0\cdot\infty$, difference: $\infty-\infty$, power: $0^\infty, \infty^0, 1^\infty$
- l'Hospital's Rule works on indeterminant quotients, convert other forms to indeterminant quotients
- Practice: 4.4.8, 4.4.13, 4.4.40, 4.4.49, 4.4.56, 4.4.71, 4.4.71, 4.4.81

4.5 Summary of Curve Sketching (week of mar 17)

- To sketch a curve $y=f(x)$ you can determine
  1. domain
  2. y-intercept ($y_{\text{int}}=f(0)$) and x-intercept if you can solve $0=f(x_{\text{int}})$
  3. symmetry: even if $f(-x)=f(x)$, odd if $f(-x)=-f(x)$, periodic if $f(x+p)=f(x)$
  4. horizontal asymptotes of $y=L$ when: $\lim_{x\to\infty}[f(x)]=L$ or $\lim_{x\to-\infty}[f(x)]=L$
  5. vertical asymptote of $x=a$ when: $\lim_{x\to a^+}[f(x)]=\infty$ or other infinite limit
  6. slant asymptote of $y=mx+b$ when: $\lim_{x\to\infty}[f(x)-(mx+b)]=0$
  7. intervals of increase or decrease (use sign chart of $f'(x)$)
  8. possible local extrema solve $f'(x)=0$ and where $f'(x)$ does not exist (critical numbers)
  9. intervals of concave up/down (use sign chart of $f''(x)$)
  10. points of inflection solve $f''(x)=0$ and where $f''(x)$ does not exist (critical numbers)
- Practice: 4.5.23, 4.5.43, 4.5.51, 4.5.56, 4.5.67

4.7 Optimization (week of mar 24)

- Think of these as finding extrema word problems, which are solved by
  1. Draw a diagram (this usually helps) and introduce notation for all quantities.
  2. Express any given information and the quantity you want to optimize (call it Q for now) using your notation.
  3. From your diagram, determine an equation for Q. This equation will not have derivatives in it.
  4. Eliminate variables if necessary so Q is a function of only one variable, $Q = f(x)$. Determine the domain.
  5. Find the absolute extrema by computing $Q' = \frac{d}{dx}[f(x)]$ and determining where $Q' = 0$ and does not exist (critical numbers).
  6. Determine if you found a min or max using a sign chart or the second derivative test. For closed domains, check endpoints and use Closed Interval Method.
- Practice: 4.7.11, 4.7.17, 4.7.31, 4.7.37, 4.7.42, 4.7.43

4.8 Newton's Method (week of mar 24)

- Newton's method approximates a root of $y = f(x)$ using $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ given x_0.
- Newton's method can fail if $f'(x_n) = 0$ (division by zero), or if you get stuck in a cycle.
- Practice: 4.8.17, 4.8.24, 4.8.31, 4.8.37, 4.8.39

4.9 Antiderivatives (week of mar 31)

- F is an antiderivative of f on interval I if $F'(x) = f(x)$ for all $x \in I$.
- The most general antiderivative is $F(x) + C$ where C is an arbitrary constant
- Practice: 4.9.11, 4.9.13, 4.9.44, 4.9.61

5.1 Areas and Distances (week of mar 31)

- $\Delta x = \frac{b-a}{n}$
- $R_n = \sum_{i=1}^{n} f(x_i^*)\Delta x$, where $x_i^* = a + \frac{b-a}{n} i$
- $L_n = \sum_{i=1}^{n} f(x_i^*)\Delta x$, where $x_i^* = a + \frac{b-a}{n}(i-1)$
- $M_n = \sum_{i=1}^{n} f(x_i^*)\Delta x$, where $x_i^* = a + \frac{b-a}{n}\left(i - \frac{1}{2}\right)$
- The Area Problem: $A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$.
- The Distance Problem using distance = velocity × time
- Practice: 5.1.3, 5.1.5, 5.1.21, 5.1.25

5.2 The Definite Integral (week of apr 7)

- Definite integral $\int_a^b f(x)dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$ where $\Delta x = \frac{b-a}{n}$
- Riemann sum $\sum_{i=1}^{n} f(x_i^*)\Delta x$
- net area: area above x-axis is positive, area below the x-axis is negative
- Theorem: If f is continuous on $[a, b]$ or if f has only a finite number of jump discontinuities, then f is integrable on $[a, b]$. That is, $\int_a^b f(x)dx$ exists.
- Evaluating integrals using

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \text{ and } \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \text{ and } \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$\sum_{i=1}^{n} c = nc$
$\sum_{i=1}^{n} ca_i = c\sum_{i=1}^{n} a_i$
$\sum_{i=1}^{n}(a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i$
$\sum_{i=1}^{n}(a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i$

- Properties of the integral (c is a constant)
  $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$
  $\int_a^a f(x)\,dx = 0$
  $\int_a^b cdx = c(b-a)$
  $\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$
  $\int_a^b [f(x)-g(x)]dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$
  $\int_a^b [cf(x)]dx = c\int_a^b f(x)\,dx$
  $\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = c\int_a^b f(x)\,dx$
- *Mathematica* `Integrate`
- Practice: 5.2.21, 5.2.31, 5.2.37, 5.2.47

5.3 The Fundamental Theorem of Calculus (WEEK OF APR 7)

- $\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$
- $\int_a^b f(x)\,dx = F(b) - F(a)$ where F is any antiderivative of f.
- Practice: 5.3.8, 5.3.19, 5.3.31, 5.3.32, 5.3.43, 5.3.53, 5.3.62

5.4 Indefinite Integrals and Net Change Theorem (WEEK OF APR 7)

- table of indefinite integrals (memorize these basic integration formulas)
- Net Change Theorem: $\int_a^b F'(x)\,dx = F(b) - F(a)$
- Practice: 5.4.1, 5.4.9, 5.4.21, 5.4.30

5.5 Substitution Rule (WEEK OF APR 14)

- $\int f(g(x))g'(x)\,dx = \int f(u)\,du$ where $u = g(x)$
- allows you to do integrals that are more complicated than the basic form integrals by transforming the integrals into a basic form (important concept)
- If f is even then $\int_{-a}^{a} f(x) = 2\int_0^a f(x)\,dx$ (prove)
- If f is odd then $\int_{-a}^{a} f(x) = 0$ (prove)
- Practice: 5.5.1, 5.5.3, 5.5.14, 5.5.30, 5.5.53, 5.5.59, 5.5.68, 5.5.69, 5.5.75, 5.5.81, 5.5.84

10.1 Parametric Equations (WEEK OF APR 14)

- the definition of parametric equations and techniques of sketching
- parametric curves
- the conversion from parametric equations to functions
- a curve has multiple parameterizations
- *Mathematica* `ParametricPlot` and `Eliminate`
- Practice: 10.1.3, 10.1.16, 10.1.24, 10.1.28

10.2 Tangents and Areas (not arclength and surface area) (WEEK OF APR 14)

- tangent to parametric curve, $\frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)}$
- second derivative $\frac{d^2y}{dx^2} = \frac{d}{dx}[\frac{dy}{dx}] = \frac{\frac{d}{dt}[\frac{dy}{dx}]}{\frac{dx}{dt}}$
- area for parametric curve $x = f(t)$, $y = g(t)$, $\alpha \le t \le \beta$ is $A = \int_a^b y\, dx = \int_\alpha^\beta g(t)f'(t)\, dt$
- Practice: 10.2.5, 10.2.7, 10.2.14, 10.2.15, 10.2.25, 10.2.33

Handout: Surfaces and Traces (WEEK OF APR 21)

- surface in $\mathbb{R}^3$ is $z = f(x, y)$
- a trace holds either x or y fixed, and creates a family of curves in the remaining two dimensions by cross sections:
  trace in xz-plane is $z = f(x, k)$
  trace in yz-plane is $z = f(k, y)$
- *Mathematica* `Plot3D`
- Practice: see Handout

Handout: Space Curves and Contour Plots (WEEK OF APR 21)

- a contour plot is just the trace where $z = k$ is fixed, and creates a family of curves in the xy-plane:
  contour plot is $k = f(x, y)$
- a space curve in $\mathbb{R}^3$ is $x = f(t),\ y = g(t)\ z = h(t)$ with $\alpha \le t \le \beta$
- limits and continuity in higher dimensions $\lim_{(x,y)\to(a,b)} f(x, y) = L$
- *Mathematica* `ContourPlot` and `ParametricPlot3D`
- Practice: see Handout

Handout: Partial Derivatives (WEEK OF APR 21)

- partial derivative with respect to x is $\dfrac{\partial z}{\partial x} = \lim_{h\to 0} \dfrac{f(x+h, y) - f(x, y)}{h}$
- partial derivative with respect to y is $\dfrac{\partial z}{\partial y} = \lim_{h\to 0} \dfrac{f(x, y+h) - f(x, y)}{h}$
- fix (hold constant) variable you are not differentiating with respect to, and use derivative rules
- higher partial derivatives
- application: error analysis
- Practice: see Handout

Handout: Extrema (WEEK OF APR 28)

- local extrema occur where $\dfrac{\partial z}{\partial x} = 0$ and $\dfrac{\partial z}{\partial y} = 0$ (system of equations)
- Practice: see Handout