1. Understand the problem. Read and think about what is happening.
2. Draw a diagram.
3. Introduce notation (Q is to be maximized or minimized)
4. Find relation between quantities (Q and all others)
5. Make the relation look like $Q = f(x)$ (one variable)
6. Figure out the domain (closed? or open?)
7. Find the absolute max or min of Q

Always make sure to include in your solution how you know you have found a max or a min (Closed Interval Method, 1st Derivative Test, 2nd Derivative Test (good one to use!), or geometry).

1. A farmer has 2400 ft of fencing. What are dimensions of rectangle that produce largest enclosed area?

2. A farmer wants to build a rectangular pen with area 600 ft^2, where one side of the pen is against a barn. What are the dimensions of the pen that minimize the amount of fencing to buy? How much fencing should be purchased?

3. Find two numbers whose sum is 23 and whose product is a maximum.

4. Find the point on the line $y = 4x + 7$ that is closest to the point $(0, 0)$.

5. A box with a square base and open top must have a volume of 32,000 cm^3. Find the dimensions of the box that minimize the amount of material used.

6. Find the point on the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.

7. A cone shaped drinking cup is made from a circular piece of paper of radius R by cutting out a sector and joining the edges. Find the maximum capacity of such a cup.

8. A rain gutter is constructed from a flat metal sheet of width 30cm by bending up 1/3 of a sheet on each side by an angle θ. What must θ be to maximize the amount of water the gutter will hold?

9. For a fish swimming at speed v relative to the water, the energy expended per unit time is proportional to v^3,

$$\frac{\text{Energy}}{\text{time}} = av^3.$$

If the current has speed u, then the speed of a fish moving against the current is $v - u$. Since

$$\text{avg speed} = \frac{\text{distance}}{\text{time}} \longrightarrow \text{time} = \frac{\text{distance}}{\text{avg speed}},$$

the time it will take the fish to travel a distance L is $L/(v - u)$.

The energy the fish expends in this time is

$$\text{Energy} = E(v) = \frac{aLv^3}{v - u}.$$

What speed should the fish travel to minimize the energy expended?

10. A rectangular piece of glass is being carried down a hallway that is 10 ft wide. At the end of the hallway there is a right angle turn and the hallway narrows to 8ft. What is the longest piece of glass that can be carried (keeping it horizontal) around the turn in the hallway?