You should be *expanding* this study guide as you see fit with details and worked examples. With this extra layer of detail you will then have excellent study notes for exams, and later reference.

Note that we will not be covering these sections in strictly numerical order, and some topics will be emphasized more than others.

Practice is suggested from Stewart *Calculus, early transcendentals*, 6th Edition.

Mathematica

- The library of built-in *Mathematica* commands we understand and use will grow throughout the semester.
- Keep a list of the basic commands we use and practice using *Mathematica*!
- You want to become proficient at typesetting text and headings in *Mathematica*.
- Use the *Mathematica Quick Reference* which was handed out in class, and use *Mathematica* to check homework whenever possible.

6.1 Areas Between Curves (WEEK OF AUG 31)

- curves that cross each other, y_T and y_B
- integration with respect to x: $\int_a^b (y_T - y_B)dx$
- integration with respect to y: $\int_c^d (x_R - x_L)dy$
- Practice: 6.1.11, 6.1.15, 6.1.39, 6.1.49

6.2 Volumes (WEEK OF SEP 7)

- creating sketches of regions
- building volume formulas from sketches using washers with $\pi[(\text{outer radius})^2 - (\text{inner radius})^2]$
- application of the general formula to other solids that aren't found via revolution (pyramids)
- Practice: 6.2.5, 6.2.13, 6.2.16, 6.2.35, 6.2.48

6.3 Volumes by Cylindrical Shells (WEEK OF SEP 7)

- creating sketched of regions
- building volume formulas from sketches using cylindrical shells with $2\pi(\text{radius})(\text{height})$
- Practice: 6.3.13, 6.3.35, 6.3.39, 6.3.43, 6.3.45, 6.3.46

6.4 Work (WEEK OF SEP 28)

- Work = Force × Distance
- Force = weight density × length or Force = mass × gravity
- Hooke's Law
- build integration formulas when
  - force varies as position changes
  - object changes shape somehow
- Practice: 6.4.10, 6.4.13, 6.4.15, 6.4.20, 6.4.18

6.5 Average Value of a Function (WEEK OF SEP 7)

- the definition of an average value $f_{\text{ave}} = \frac{1}{b-a}\int_a^b f(x)dx$
- a geometric interpretation of average value
- Mean Value Theorem $\int_a^b f(x)dx = (b-a)f(c)$ for $c \in [a,b]$
- Practice: 6.5.11, 6.5.19, 6.5.22

7.1 Integration by Parts (WEEK OF SEP 14)

- the formula for integration by parts $\int u dv = uv - \int v du$
- substitution and integration by parts
- multiple applications of integration by parts
- parts, parts, algebra for forms like $\int e^x \cos x dx$
- $\int \ln x \, dx = x \ln x - x + c$
- Practice: 7.1.18, 7.1.19, 7.1.29, 7.1.59, 7.1.67

7.2 Trigonometric Integrals (WEEK OF SEP 14)

- trigonometric identities

  $\sin^2 x + \cos^2 x = 1$
  $\sin^2 x = \frac{1}{2}(1 - \cos(2x))$
  $\cos^2 x = \frac{1}{2}(1 + \cos(2x))$
  $\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)]$ and similar
- strategies for integrals of products of sine and cosine
- strategies for integrals of products of tangent and secant
- $\int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx = \tan x - x + c$
- $\int \tan x \, dx = \ln|\sec x| + c$
- $\int \sec x \, dx = \ln|\sec x + \tan x| + c$
- Practice: 7.2.4, 7.2.12, 7.2.13, 7.2.41, 7.2.67

7.3 Trigonometric Substitution (WEEK OF SEP 14)

- inverse substitution
- trig identities

  $\cos^2\theta + \sin^2\theta = 1$ which leads to
  $1 - \sin^2\theta = \cos^2\theta$
  $\sec^2\theta - 1 = \tan^2\theta$
  $1 + \tan^2\theta = \sec^2\theta$
- choosing correct trig substitution

  $\sqrt{a^2 - x^2} \longrightarrow x = a\sin\theta$
  $\sqrt{x^2 - a^2} \longrightarrow x = a\sec\theta$
  $\sqrt{a^2 + x^2} \longrightarrow x = a\tan\theta$
- completing the square
- Practice: 7.3.5, 7.3.18, 7.3.34, 7.3.40, 7.3.41

7.4 Integration of Rational Functions by Partial Fractions (WEEK OF SEP 21)

- long division of polynomials
- three cases of partial fraction expansion
  - distinct linear factors $\frac{1}{(x-3)(x-4)} = \frac{A}{x-3} + \frac{B}{x-4}$
  - repeated linear factors $\frac{1}{(x-3)(x-4)^2} = \frac{A}{x-3} + \frac{B}{(x-4)} + \frac{C}{(x-4)^2}$
  - distinct irreducible quadratic factors $\frac{1}{(x^2-x+3)(x-4)^2} = \frac{Ax+B}{x^2-x+3} + \frac{C}{(x-4)} + \frac{D}{(x-4)^2}$
- *Mathematica* command `Apart`
- rationalizing substitution
- Practice: 7.4.9, 7.4.23, 7.4.33, 7.4.39, 7.4.65

7.5 Strategy for Integration (WEEK OF SEP 21)

- memorize basic integral formulas
- know the reasons for trying different integration techniques
- many integrals can be done more than one way
- check in *Mathematica* using command `Integrate`
- Practice: 7.5.5, 7.5.37, 7.5.40, 7.5.44, 7.5.73

7.6 Integration using Tables and CAS (WEEK OF SEP 28)

- making your integral look like one in a table
- things to be aware of when using CAS
- *Mathematica* commands with assumptions `Integrate[1/x^r,{x,0,4},Assumptions->{r<1}]`
- Practice: 7.6.43, 7.6.44

7.7 Approximate Integration (WEEK OF SEP 28)

- midpoint rule
- trapezoidal rule
- Simpson's rule
- error bounds
- how to implement in *Mathematica* (for data points and when $f(x)$ given)
- Practice: 7.7.19, 7.7.22, 7.7.31, 7.7.40

7.8 Improper Integrals (WEEK OF SEP 28)

- integration over infinite intervals using limits such as $\int_0^\infty e^{-x}dx = \lim_{w\to\infty} \int_0^w e^{-x}dx$
- intervals over discontinuous integrands using limits such as $\int_0^1 \ln x dx = \lim_{w\to 0^+} \int_w^1 \ln x dx$
- convergence and divergence of such integrals, and ones like $\int_1^\infty 1/x^p dx$
- comparison test
- Practice: 7.8.1, 7.8.17, 7.8.27, 7.8.66, 7.8.78

8.3 Applications to Physics and Engineering (WEEK OF OCT 5)

- moments and centers of mass $(\bar{x}, \bar{y})$

  $\bar{x} = \frac{1}{m}\int_a^b x(f(x) - g(x))dx$
  $\bar{y} = \frac{1}{m}\int_a^b \frac{1}{2}\left([f(x)]^2 - [g(x)]^2\right)dx$
- centroids ($\rho = 1$)
- Theorem of Pappus
- Practice: 8.3.27, 8.3.31, 8.3.33, 8.3.34, 8.3.38

8.4 Applications to Economics and Biology (WEEK OF OCT 5)

- consumer and producer surplus
- blood flow
- cardiac output
- AUC for Blood Plasma
- Practice: 8.4.5, 8.4.8, 8.4.13, 8.4.17

8.5 Probability (WEEK OF OCT 5)

- probability density function (PDF) $\int_{-\infty}^{\infty} f(t)dt = 1$
- average values $\mu = \int_{-\infty}^{\infty} tf(t)dt$
- median value m satisfies $\int_{-\infty}^{m} f(t)dt = \frac{1}{2}$
- most probable value is $\frac{d}{dt}f(t) = 0$
- normal distributions and standard deviation
- wait time PDF $f(t) = \mu^{-1}e^{-t/\mu}, t \geq 0$
- Practice: 8.5.10, 8.5.11, 8.5.15, 8.5.19

9.1 Modeling with Differential Equations (WEEK OF NOV 16)

- models of population growth
- carrying capacity
- spring motion
- general differential equations (DE) and initial value problems (IVP)
- Practice: 9.1.2, 9.1.3, 9.1.11, 9.1.13

9.2 Direction Fields and Euler's Method (WEEK OF NOV 16)

- direction fields provide graphical solutions to differential equations
- Euler's method provides numerical solutions to initial value problems
- implementing both in *Mathematica*
- Practice: 9.2.3-6, 9.2.25, 9.2.28

9.3 Separable Equations (week of nov 23)

- separable equations can be solved by direct integration $dy/dx = f(x)/g(y)$
- an orthogonal trajectory to a family of curves intersects each curve of the family orthogonally
- mixing problems
- Practice: 9.3.5, 9.3.7, 9.3.29, 9.3.37, 9.3.39, 9.3.41

9.4 Models for Population Growth (week of nov 23)

- exponential growth $dP/dt = kP$, $k > 0$
- logistic equation $dP/dt = kP(1 - P/K)$, $k, K > 0$
- Gompertz equation $dP/dt = cP\ln(K/P)$, $c, K > 0$
- doomsday equation $dP/dt = kP^{1+c}$, $c > 0$
- Practice: 9.4.1, 9.4.3, 9.4.13, 9.4.19

9.5 Linear Equations (week of nov 30)

- linear first order $dy/dt + P(x)y = Q(x)$
- integrating factor method, $I(x) = e^{\int P(x)dx}$
- Practice: 9.5.3, 9.5.10, 9.5.17, 9.5.23, 9.5.33, 9.3.35

9.6 Predator-Prey Systems (week of nov 30)

- system of differential equations
- phase plane
- cooperation vs competition
- equilibrium solutions, solve $dx/dt = 0, dy/dt = 0$
- solving system by eliminating a variable $dx/dt = f, dy/dt = g \longrightarrow dx/dy = f/g$
- numerical solutions using *Mathematica*
- Practice: 9.6.2, 9.6.7, 9.6.8, 9.6.9

10.1 Curves Defined by Parametric Equations (week of nov 30)

- the definition of parametric equations and techniques of sketching
- parametric curves
- *Mathematica* `ParametricPlot[{Sin[4*t]*Cos[t],Cos[t]},{t,-Pi,Pi}]`
- the conversion from parametric equations to functions
- a curve has multiple parameterizations
- Practice: 10.1.3, 10.1.16, 10.1.24, 10.1.28

10.2 Calculus with Parametric Curves (week of nov 30)

- tangent to parametric curve, $dy/dx = (dy/dt)/(dx/dt)$; second derivative d^2y/dx^2
- arc length of a parametric curve; the meaning of $(ds)^2 = (dx)^2 + (dy)^2$
- formulation of an integral to compute the arc length $L = \int_a^b ds$
- formulation of an integral to compute surface area of revolution $S = \int_\alpha^\beta 2\pi y\, ds$

- Practice: 10.2.41, 10.2.45, 10.2.61, 10.3.74

10.3 Polar Coordinates (WEEK OF DEC 7)

- the geometric and algebraic definitions of the polar coordinates r and θ
- conversion between rectangular and polar coordinates
- graphing a polar function in *Mathematica* `PolarPlot[Sin[2.4 t]^2+Cos[2.4 t]^4,{t,0,10Pi}]`
- Practice: 10.3.19, 10.3.25, 10.3.39

10.4 Areas and Lengths in Polar Coordinates (WEEK OF DEC 7)

- the computation of the area of a region bounded by polar curves
- the computation of the arc length of a polar curve
- Practice: 10.4.9, 10.4.29, 10.4.37

11.1 Sequences (WEEK OF OCT 12)

- Zeno's paradoxes
- the basic definition of a sequence $\{a_n\}$
- the difference between $\{a_n\}$ (discrete) and $f(x)$ (continuous) (important if using l'Hospital's rule)
- the limit of a sequence $\lim_{n\to\infty} a_n$
- increasing, decreasing, monotonic, bounded above, bounded below, bounded
- the Monotonic Sequence Theorem
- mathematical induction
- recursive sequences $a_n = f(a_{n-1})$
- Practice: 11.1.23, 11.1.41, 11.1.51, 11.1.67, 11.1.69

11.2 Series (WEEK OF OCT 19)

- basic concept of a series $s = \sum_{i=1}^{\infty} a_i$
- difference between the underlying sequence $\{a_n\}$ and the sequence of partial sums $s_n = \sum_{i=1}^{n} a_i$
- convergence of series is determined by evaluating the limit of sequence of partial sums $s = \lim_{n\to\infty} s_n$
- relatively few series we can determine sum exactly, including geometric and telescoping series
- properties and uses of geometric series $\sum_{n=1}^{\infty} r^{n-1} = \frac{1}{1-r}$ if $|r| < 1$
- telescoping series such as $\sum_{i=1}^{n} \frac{1}{i(i+1)} = 1 - \frac{1}{n+1}$ (use partial fractions)
- test for Divergence: if $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ is divergent
- Practice: 11.2.25, 11.2.27, 11.2.44, 11.2.49, 11.2.53, 11.2.59

11.3 The Integral Test and Estimates of Sums (WEEK OF OCT 26)

- determine if a series converges or diverges, when we cannot find the sum exactly
- the geometry and formal statement of the integral test
- the remainder estimate for the integral test
- the p-series $\sum_{n=1}^{\infty} 1/n^p$ converges if $p > 1$
- Practice: 11.3.7, 11.3.20, 11.3.21, 11.3.29, 11.3.42

11.4 The Comparison Tests (week of oct 26)

- the comparison test

  if $\sum b_n$ is convergent and $a_n \le b_n$ for all n, then $\sum a_n$ is also convergent
  if $\sum b_n$ is divergent and $a_n \ge b_n$ for all n, then $\sum a_n$ is also divergent
- we need only consider the "tail" of the series to determine convergence or divergence
- the limit comparison test for $a_n > 0$ and $b_n > 0$ for all n

  $\lim_{n\to\infty} a_n/b_n = c > 0$ (finite), then either both $\sum a_n$ and $\sum b_n$ converge, or both diverege
- good comparison series are p-series and geometric series
- Practice: 11.4.7, 11.4.9, 11.4.19, 11.4.39, 11.4.40

11.5 Alternating Series (week of oct 26)

- the alternating series test: for $b_n > 0$, if

  (i) $b_{n+1} \le b_n$ and (ii) $\lim_{n\to\infty} b_n = 0$ then the series$\sum(-1)^{n-1}b_n$ converges
- the alternating series estimation theorem
- Practice: 11.5.15, 11.5.19, 11.5.33

11.6 Absolute Convergence and the Ratio and Root Tests (week of oct 26)

- the series $\sum a_n$ is absolutely convergent if $\sum |a_n|$ converges
- the series $\sum a_n$ is conditional convergent if it is convergent but not absolutely convergent (example $a_n = (-1)^n/n$)
- the ratio test: $\lim_{n\to\infty} |\frac{a_{n+1}}{a_n}| = L$

  (i) $L < 1$ $\sum a_n$ absolutely convergent (ii) $L > 1$ $\sum a_n$ diverges (iii) $L = 1$ test is inconclusive
- the root test: $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L$

  (i) $L < 1$ $\sum a_n$ absolutely convergent (ii) $L > 1$ $\sum a_n$ diverges (iii) $L = 1$ test is inconclusive
- if ratio test is inconclusive, the root test is inconclusive
- rearrangements: if $\sum a_n$ is absolutely convergent, rearrangements give the same sum
- Practice: 11.6.5, 11.6.13, 11.6.14, 11.6.21, 11.6.24, 11.6.38

11.7 Strategy for Testing Series (week of nov 2)

- make intelligent guesses as to convergence/divergence to determine which test to try
- often, more than one test can be used
- know the series which converge/diverge and look for similarities (p-series, geometric series, etc)
- it is better to try than not to try (as in life, so be it for series)
- Practice: 11.7.8, 11.7.13, 11.7.17, 11.7.23, 11.7.24, 11.7.36

11.8 Power Series (week of nov 2)

- the definition of a power series $\sum_{n=0}^{\infty} c_n(x-x_0)^n$
- the radius of convergence of a power series (use ratio test)
- the interval of convergence (use a different test to check endpoints of interval)
- Practice: 11.8.11, 1..8.13, 11.8.19, 11.8.31, 11.8.33, 11.8.40

11.9 Representations of Functions as Power Series (week of nov 2)

- term-by-term differentiation and integration of power series (inside radius of convergence)
- use geometric series to get new power series from old

  $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ converges for $|x| < 1$
  substitution $x \to -2x$: $\frac{1}{1+2x} = \sum_{n=0}^{\infty}(-1)^n 2^n x^n$ converges for $|x| < 1/2$
  $\frac{d}{dx}\ln(1-x) = -\frac{1}{1-x} = -\sum_{n=0}^{\infty} x^n$ so integrate to get series for $\ln(1-x)$
  $\frac{d}{dx}\arctan x = \frac{1}{1+x^2} = \sum_{n=0}^{\infty}(-1)^n x^{2n}$ so integrate to get series for $\arctan x$
- Practice: 11.9.7, 11.9.9, 11.9.14, 11.9.15, 11.9.18, 11.9.30, 11.9.39

11.10 Taylor and MacLaurin Series (week of nov 9)

- Taylor series for f about a is $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$
- MacLaurin series for f is $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$ (ie. $a = 0$)
- kth degree Taylor polynomial for f about a is $T_k(x) = \sum_{n=0}^{k} \frac{f^{(n)}(a)}{n!}(x-a)^n$ (truncate infinite series)
- the remainder $R_n(x) = f(x) - T_n(x)$
- Taylor's inequality
- binomial series $(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n = 1 + kx + \frac{k(k-1)}{2!} + \frac{k(k-1)(k-2)}{3!} + \cdots$
- common MacLaurin series (you should know how to get these and have them memorized)

  $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$
  $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$
  $\sin x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$
  $\cos x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}$
  $\arctan x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{2n+1}$
  $(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n$
- multiplication and division of power series
- Practice: 11.10.16, 11.10.17, 11.10.27, 11.10.33, 11.10.37, 11.10.51, 11.10.58, 11.10.64

11.11 Application of Taylor Polynomials (week of nov 9)

- numerical examples (approximations to functions, approximate integration)
- theoretical examples
- Practice: 11.11.32, 11.11.36, 11.11.37