Special Case of Integration: When the Integrand is 1

When the integrand is one, the integral is easy to evaluate:

$$\int 1\,dx = \int dx = x + C.$$

This is true since $\frac{d}{dx}[x+C] = 1$ (we have found an antiderivative of the integrand). Think of this as adding up small bits of x to get all of x, (remember, dx can be thought of as a small bit of x, and integration is the process of adding things up).

This leads to impressive results like the following:

$$\begin{aligned}\int dy &= y + C,\\ \int dz &= z + C,\\ \int d\heartsuit &= \heartsuit + C.\end{aligned}$$

The most general and useful way to say this is as follows:

$$\int d[f(x)] = f(x) + c.$$

Deriving the Indefinite Integral Formulas from the Basic Derivative Formulas

We can now work out the integral formulas from the table on page 406 of the text. All it requires is that we know the derivative formulas that leads to them! <u>This is how I remember the basic integral formulas</u>. For example,

$$\begin{aligned}\frac{d}{du}[\sin u] &= \cos u \text{ (start with a known derivative)}\\ d[\sin u] &= \cos u\,du \text{ (rewrite)}\\ \int d[\sin u] &= \int \cos u\,du \text{ (integrate both sides–powerful use of notation!)}\\ \sin u &= \int \cos u\,du \text{ (use result from above when integrand is 1)}\\ \int \cos u\,du &= \sin u + C \text{ (rewrite, and include constant)}\end{aligned}$$

With this process in mind, you can easily (and quickly) work out the basic integral formulas from the basic derivative formulas whenever you need them! This saves having to memorize two very similar things (derivatives and integrals). Of course, the basic derivative formulas still need to be memorized!

Here is another example (this is the most involved of the lot, since it involves relabeling at the end):

$$\begin{aligned}\frac{d}{du}[u^n] &= nu^{n-1} \text{ (start with a known derivative)}\\ d[u^n] &= nu^{n-1}\,du \text{ (rewrite)}\\ \int d[u^n] &= \int nu^{n-1}\,du \text{ (integrate both sides)}\\ u^n &= n\int u^{n-1}\,du \text{ (use result from above when integrand is 1)}\\ \int u^{n-1}\,du &= \frac{1}{n}u^n + C,\ \ n \neq 0 \text{ (rewrite, and include constant, exclude } n=0 \text{ to avoid division by zero)}\\ \int u^m\,du &= \frac{1}{m+1}u^{m+1} + C,\ \ m \neq -1 \text{ (relabel, let } m = n-1)\end{aligned}$$

The Other Important Basic Integral Formulas

$$\begin{aligned}
\frac{d}{du}[e^u] &= e^u \text{ (known derivative)} \\
d[e^u] &= e^u\,du \\
\int d[e^u] &= \int e^u\,du \\
e^u &= \int e^u\,du \\
\int e^u\,du &= e^u + C
\end{aligned}$$

$$\begin{aligned}
\frac{d}{du}[\cos u] &= -\sin u \text{ (known derivative)} \\
d[\cos u] &= -\sin u\,du \\
\int d[\cos u] &= -\int \sin u\,du \\
\cos u &= -\int \sin u\,du \\
\int \sin u\,du &= -\cos u + C
\end{aligned}$$

$$\begin{aligned}
\frac{d}{du}[\tan u] &= \sec^2 u \text{ (known derivative)} \\
d[\tan u] &= \sec^2 u\,du \\
\int d[\tan u] &= \int \sec^2 u\,du \\
\tan u &= \int \sec^2 u\,du \\
\int \sec^2 u\,du &= \tan u + C
\end{aligned}$$

$$\begin{aligned}
\frac{d}{du}[\cot u] &= -\csc^2 u \text{ (known derivative)} \\
d[\cot u] &= -\csc^2 u\,du \\
\int d[\cot u] &= -\int \csc^2 u\,du \\
\cot u &= -\int \csc^2 u\,du \\
\int \csc^2 u\,du &= -\cot u + C
\end{aligned}$$

$$\begin{aligned}
\frac{d}{du}[\sec u] &= \sec u \tan u \text{ (known derivative)} \\
d[\sec u] &= \sec u \tan u\,du \\
\int d[\sec u] &= \int \sec u \tan u\,du \\
\sec u &= \int \sec u \tan u\,du \\
\int \sec u \tan u\,du &= \sec u + C
\end{aligned}$$

$$\begin{aligned}
\frac{d}{du}[\ln|u|] &= \frac{1}{u} \text{ (known derivative)} \\
d[\ln|u|] &= \frac{1}{u}\,du \\
\int d[\ln|u|] &= \int \frac{1}{u}\,du \\
\ln|u| &= \int \frac{1}{u}\,du \\
\int \frac{1}{u}\,du &= \ln|u| + C
\end{aligned}$$

$$\begin{aligned}
\frac{d}{du}[\csc u] &= -\csc u \cot u \text{ (known derivative)} \\
d[\csc u] &= -\csc u \cot u\,du \\
\int d[\csc u] &= -\int \csc u \cot u\,du \\
\csc u &= -\int \csc u \cot u\,du \\
\int \csc u \cot u\,du &= -\csc u + C
\end{aligned}$$

$$\begin{aligned}
\frac{d}{du}[\arctan u] &= \frac{1}{u^2+1} \text{ (known derivative)} \\
d[\arctan u] &= \frac{1}{u^2+1}\,du \\
\int d[\arctan u] &= \int \frac{1}{u^2+1}\,du \\
\arctan u &= \int \frac{1}{u^2+1}\,du \\
\int \frac{1}{u^2+1}\,du &= \arctan u + C
\end{aligned}$$

$$\begin{aligned}
\frac{d}{du}[\arcsin u] &= \frac{1}{\sqrt{1-u^2}} \text{ (known derivative)} \\
d[\arcsin u] &= \frac{1}{\sqrt{1-u^2}}\,du \\
\int d[\arcsin u] &= \int \frac{1}{\sqrt{1-u^2}}\,du \\
\arcsin u &= \int \frac{1}{\sqrt{1-u^2}}\,du \\
\int \frac{1}{\sqrt{1-u^2}}\,du &= \arcsin u + C
\end{aligned}$$

You can do this for other derivatives you know, but these are typically the ones that arise most often.