Sequences

A sequence $\{a_1, a_2, a_3, \ldots, a_n, \ldots\}$ is denoted $\{a_n\}$ and the sequence converges if $\lim_{n\to\infty} a_n$ exists and is finite.

- We can try to find the limit of the sequence by evaluating $\lim_{n\to\infty} a_n$ using limit laws, and remembering that we need to work with the function $f(x)$ where $f(n) = a_n$ if we need to use l'Hospital's rule since derivatives are only defined over a continuous variable (and n is a discrete variable).
- In some cases, we can use the Monotonic Sequence Theorem to find out if the sequence converges. This won't tell us what the limit is, but can tell us if the limit exists. This technique is useful if the sequence is given recursively (induction is useful for recursively defined sequences).

Series

A series is constructed from a sequence $\{a_n\}$ via:

$$\sum_{i=1}^{\infty} a_i = a_1 + a_2 + a_3 + \cdots.$$

A series is classified as either convergent or divergent.

If the series converges, then the sequence of partial sums $\{s_n\}$ converges, and we have a sum s for the series:

$$s_n = \sum_{i=1}^{n} a_i \qquad\qquad \lim_{n\to\infty} s_n = \sum_{i=1}^{\infty} a_i = a_1 + a_2 + a_3 + \cdots = s.$$

- Sometimes we can find s exactly by evaluating $\lim_{n\to\infty} s_n$:
  - geometric series
  - telescoping series
  - other series on a case-by-case basis (like 11.2.67, where we showed that $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n+1)!} = 1$.)
- Sometimes we cannot find s exactly, and we have to rely on various tests to determine if $\displaystyle\sum_{i=1}^{\infty} a_i$ converges. The tests do not tell us what the sum s is, but can tell us if the series is convergent or divergent.
- The tests we use are:
  - The Test for Divergence
  - The Integral Test
  - The Comparison Test
  - The Limit Comparison Test
  - The Alternating Series Test
  - The Root Test
  - The Ratio Test