

Name: \_\_\_\_\_

## Instructions:

- **Answer 8 out of the 10 problems.** There will be no bonus marks for attempting more than 8 problems. Before you hand in your exam, tell me the two problems you do not want graded:

**Do not grade problems \_\_\_\_\_ and \_\_\_\_\_.**

- The exam is out of 200 marks.
- You have 120 minutes to complete the exam (15 minutes per question).
- You must work alone, without *Mathematica* or calculators.
- You must show the procedure you used to solve the problem.
- Good Luck!

## Useful Relations:

- Trigonometric Identities:

$$1 + \cot^2 x = \csc^2 x$$

$$\sin x \cos x = \frac{1}{2} \sin 2x$$

$$\sin(x + y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

$$\sin x \cos y = \frac{1}{2} [\sin(x + y) + \sin(x - y)]$$

$$\cos x \cos y = \frac{1}{2} [\cos(x + y) + \cos(x - y)]$$

$$\sin x \sin y = \frac{1}{2} [\cos(x + y) - \cos(x - y)]$$

$$\cos^2 x + \sin^2 x = 1$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\cos 2x = 1 - 2\sin^2 x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\sec^2 x - \tan^2 x = 1$$

- $(ds)^2 = (dx)^2 + (dy)^2$

- $A = \int_a^b \frac{1}{2} r^2 d\theta$

- $L = \int_a^b \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$

- Geometric series  $\sum_{n=1}^{\infty} ar^{n-1}$  converges to  $a/(1-r)$  if  $|r| < 1$

- The  $p$ -series  $\sum_{n=1}^{\infty} 1/n^p$  is convergent if  $p > 1$

- $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n, \quad |x| < 1$