Math 1102 Calculus II
Quiz 2: Thursday Feb 1, 2007 **Barry's Solution**

Instructions: You have 15 minutes, you may not use *Mathematica*, and you must work alone, but you can use your notes and text.

1. (20 marks) Evaluate the integral $I = \int \frac{dt}{\sqrt{t^2 + 2t - 8}}$.

Solution This looks like a trig substitution, since there is a square root in the integrand. However, it needs some modification before we can integrate.

First, complete the square:

$$t^2 + 2t = (t+1)^2 - 1$$

$$t^2 + 2t - 8 = (t+1)^2 - 9$$

The integrand becomes

$$\begin{aligned} I &= \int \frac{dt}{\sqrt{t^2 + 2t - 8}} \\ &= \int \frac{dt}{\sqrt{(t+1)^2 - 9}} \quad \text{Substitution: } \begin{array}{l} x = t+1 \\ dx = dt \end{array} \\ &= \int \frac{dx}{\sqrt{x^2 - 9}} \end{aligned}$$

This is now ready for a trig substitution. The integral contains a $\sqrt{x^2 - a^2}$, with $a = 3$, so we should use the trig substitution:

$$\begin{aligned} x &= a \sec\theta = 3\sec\theta \\ dx &= 3\sec\theta\tan\theta\, d\theta \end{aligned}$$

Now, we find expressions for all the components of the integrand:

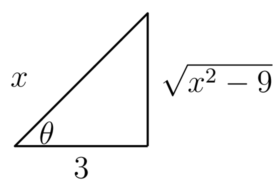
$$\begin{aligned} \sqrt{x^2 - 9} &= \sqrt{9\sec^2\theta - 9} \\ &= \sqrt{9}\sqrt{\sec^2\theta - 1} \\ &= 3\sqrt{\tan^2\theta} \\ &= 3|\tan\theta| = 3\tan\theta \end{aligned}$$

And now we do the integral:

$$\begin{aligned}
\int \frac{dx}{\sqrt{x^2-9}} &= \int \frac{3\sec\theta\tan\theta\,d\theta}{3\tan\theta} \\
&= \int \sec\theta\,d\theta \\
&= \ln|\sec\theta+\tan\theta| + C
\end{aligned}$$

Now we need to back substitute to get the integral in terms of t. First, construct the diagram that will help us back substitute the θ:

$$\sec\theta = \frac{x}{3}$$



$$\sec\theta = \frac{x}{3}, \quad \tan\theta = \frac{\sqrt{x^2-9}}{3},$$

And now we can finish the back substitution:

$$\begin{aligned}
I &= \int \frac{dt}{\sqrt{t^2+2t-8}} \\
&= \ln|\sec\theta+\tan\theta| + C \\
&= \ln\left|\frac{x}{3}+\frac{\sqrt{x^2-9}}{3}\right| + C \\
&= \ln\left|\frac{t+1}{3}+\frac{\sqrt{(t+1)^2-9}}{3}\right| + C
\end{aligned}$$

This wasn't asked, but for what values of $t > 0$ is this formula valid?

This formula will be valid for any region of t which is in the domain of the function. Therefore, $(t+1)^2 - 9 \geq 0$, or we get a square root of a negative number, which has no solution for the real numbers. This means that our formula is valid for $t \geq 2$ (since we are told that $t > 0$ for this problem).