13.1.14 $\vec{r}(t) = e^{-t}\,\hat{i} + 2\cos 3t\,\hat{j} + 2\sin 3t\,\hat{k}, \quad t=0.$

$$\vec{v}(t) = -e^{-t}\,\hat{i} - 6\sin 3t\,\hat{j} + 6\cos 3t\,\hat{k}$$

$$\vec{a}(t) = e^{-t}\,\hat{i} - 18\cos 3t\,\hat{j} - 18\sin 3t\,\hat{k}$$

$$\vec{v}(0) = -\hat{i} - 0\hat{j} + 6\hat{k} = \langle -1, 0, 6\rangle$$
$$\vec{a}(0) = \hat{i} - 18\hat{j} - 0\hat{k} = \langle 1, -18, 0\rangle$$

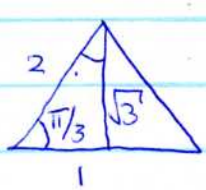
speed at $t=0$ is $|\vec{v}(0)| = \sqrt{1+36} = \sqrt{37}$

$\vec{v}(0) = \sqrt{37}\left\langle \frac{-1}{\sqrt{37}}, 0, \frac{6}{\sqrt{37}} \right\rangle$ (mag)(unit vector)

13.1.24 $\displaystyle\int_0^{\pi/3} \left[(\sec t \tan t)\,\hat{i} + \tan t\,\hat{j} + 2\sin t\cos t\,\hat{k} \right] dt$

$$= \int_0^{\pi/3} \sec t\tan t\,dt\;\hat{i} + \int_0^{\pi/3}\tan t\,dt\;\hat{j} + 2\int_0^{\pi/3}\sin t\cos t\,dt\;\hat{k}$$

$$= \sec t\Big|_0^{\pi/3}\hat{i} - \ln(\cos t)\Big|_0^{\pi/3}\hat{j} - \cos^2 t\Big|_0^{\pi/3}\hat{k}$$



$\cos \pi/3 = \frac{1}{2} \implies \sec \pi/3 = 2$ Also $\cos 0 = 1$
$\Rightarrow \sec 0 = 1$

$$= (\sec \pi/3 - \sec 0)\,\hat{i} - (\ln 1/2 - \ln 1)\,\hat{j} - \left((1/2)^2 - 1^2\right)\hat{k}$$
$$= \hat{i} - \ln 1/2\,\hat{j} + \frac{3}{4}\hat{k}$$
$$= \hat{i} + \ln 2\,\hat{j} + \frac{3}{4}\hat{k}$$
$$= \langle 1, \ln 2, 3/4\rangle.$$

13.1.38 $\vec{r}(t) = (2\hat{i} + 2\hat{j} + \hat{k}) + \cos t\left(\frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}\right) + \sin t\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)$

Get a parametric representation of the path the particle is following (we have more experience with those):

$x = 2 + \frac{1}{\sqrt{2}}\cos t + \frac{1}{\sqrt{3}}\sin t$ (from $\hat{i}$ component)

$y = 2 - \frac{1}{\sqrt{2}}\cos t + \frac{1}{\sqrt{3}}\sin t$ (from $\hat{j}$ component)

$z = 1 + \frac{1}{\sqrt{3}}\sin t$ (from $\hat{k}$ component)

Eliminate t (you could use MMA for this if you like)

$\sin t = \sqrt{3}(z-1)$

$\Rightarrow x = 2 + \frac{1}{\sqrt{2}}\cos t + (z-1)$

$y = 2 - \frac{1}{\sqrt{2}}\cos t + (z-1)$ add

$x + y = 4 + 2(z-1)$

$\Rightarrow x + y - 2z = 2$ which is a plane. So the particle is moving in a plane.

If the particle moves in a circle with radius 1 centered at $(2,2,1)$, then the distance at any t from $(2,2,1)$ should be 1. Let's check that.

13.1.38 continued

$$\begin{pmatrix}\text{distance}\\ \text{to } (2,2,1)\end{pmatrix}^2 = (x-2)^2 + (y-2)^2 + (z-1)^2$$

$$= \left(\frac{1}{\sqrt{2}}\cos t + \frac{1}{\sqrt{3}}\sin t\right)^2 + \left(\frac{-1}{\sqrt{2}}\cos t + \frac{1}{\sqrt{3}}\sin t\right)^2 + \frac{1}{3}\sin^2 t$$

$$= \cancel{\frac{2}{\sqrt{6}}\cos t \sin t} + \frac{1}{2}\cos^2 t + \frac{1}{3}\sin^2 t \cancel{- \frac{2}{\sqrt{6}}\cos t \sin t}$$

$$+ \frac{1}{2}\cos^2 t + \frac{1}{3}\sin^2 t + \frac{1}{3}\sin^2 t$$

$$= \cos^2 t + \sin^2 t$$

$$= \underline{1}$$

$\Rightarrow$ distance to $(2,2,1)$ is <u>always</u> $\underline{1}$.

13.1.43

$$\vec{r}(t) = 3\cos t\,\hat{j} + 2\sin t\,\hat{k}$$

$$\vec{v}(t) = -3\sin t\,\hat{j} + 2\cos t\,\hat{k}$$
$$\vec{a}(t) = -3\cos t\,\hat{j} - 2\sin t\,\hat{k}$$

$$|\vec{v}(t)|^2 = 9\sin^2 t + 4\cos^2 t$$
$$|\vec{a}(t)|^2 = 9\cos^2 t + 4\sin^2 t$$

Max of $|\vec{v}(t)|^2$ occurs when $\frac{d}{dt}|\vec{v}(t)|^2 = 0$

$$\Rightarrow \quad 18\sin t\cos t + 8\cos t(-\sin t) = 0$$
$$10\sin t\cos t = 0$$
$$\sin t\cos t = 0$$
$$\frac{1}{2}\sin 2t = 0 \qquad \text{use } \sin 2\theta = 2\sin\theta\cos\theta$$

$\longrightarrow \quad 2t = n\pi, \quad n = \ldots -2, -1, 0, 1, 2, \ldots$

$$t = \frac{n\pi}{2}.$$

13.1.43 continued

The question now is which gives a max, and which gives a min. Let's use the 2ND derivative test.

$$\frac{d}{dt}|\vec{v}(t)|^2 = 5\sin 2t$$
$$\frac{d^2}{dt^2}|\vec{v}(t)|^2 = 10\cos 2t.$$

At $t=0$, this is $>0 \Rightarrow |\vec{v}|^2$ concave up $\Rightarrow |\vec{v}|^2$ has a <u>min</u>.
At $t=\pi/2$, this is $<0 \Rightarrow |\vec{v}|^2$ concave down $\Rightarrow |\vec{v}|^2$ has a <u>max</u>.
Other n values just repeat this.

So $t=0$ gives a min of $|\vec{v}|^2 = 4 \Rightarrow |\vec{v}| = 2$.
$t=\pi/2$ gives a max of $|\vec{v}|^2 = 9 \Rightarrow |\vec{v}| = 3$.

Extrema of $|\vec{a}(t)|^2$ occurs when $\frac{d}{dt}|\vec{a}(t)|^2 = 0$.

$$\frac{d}{dt}|\vec{a}(t)|^2 = -5\sin 2t = 0 \Rightarrow t = n\pi/2,\ n \text{ integer.}$$
$$\ast\quad \frac{d^2}{dt^2}|\vec{a}(t)|^2 = -10\cos 2t.$$

At $t=0$, $\ast$ is $<0 \Rightarrow |\vec{a}(t)|^2$ concave down $\Rightarrow |\vec{a}(t)|^2$ has a max
$t=\pi/2$, $\ast$ is $>0 \Rightarrow |\vec{a}(t)|^2$ concave up $\Rightarrow |\vec{a}(t)|^2$ has a min.

So $t=0$ gives a max of $|\vec{a}(t)|^2 = 9 \Rightarrow |\vec{a}| = 3$
$t=\pi/2$ gives a min of $|\vec{a}(t)|^2 = 4 \Rightarrow |\vec{a}| = 2$.

13.1.54c

$$\int_a^b \vec{c} \cdot \vec{r}(t)\,dt = \int_a^b c_1 x(t)\,dt + \int_a^b c_2 y(t)\,dt + \int_a^b c_3 z(t)\,dt$$

Let $\vec{c} = \langle c_1, c_2, c_3 \rangle$

$\vec{r}(t) = \langle x(t), y(t), z(t) \rangle$

$$= c_1 \int_a^b x(t)\,dt + c_2 \int_a^b y(t)\,dt + c_3 \int_a^b z(t)\,dt$$

$$= \langle c_1, c_2, c_3 \rangle + \int_a^b \langle x(t), y(t), z(t) \rangle\,dt$$

$$= \vec{c} \cdot \int_a^b \vec{r}(t)\,dt$$

$$\int_a^b \vec{c} \times \vec{r}(t)\,dt = \int_a^b \det \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ c_1 & c_2 & c_3 \\ x(t) & y(t) & z(t) \end{vmatrix} dt$$

$$= \int_a^b \hat{i} \det\begin{pmatrix} c_2 & c_3 \\ y(t) & z(t) \end{pmatrix} dt - \int_a^b \hat{j} \det\begin{pmatrix} c_1 & c_3 \\ x(t) & z(t) \end{pmatrix} dt$$

$$+ \int_a^b \hat{k} \det\begin{pmatrix} c_1 & c_2 \\ x(t) & y(t) \end{pmatrix} dt$$

$$= \int_a^b \hat{i} \left(c_2 z(t) - c_3 y(t) \right) dt - \int_a^b \hat{j} \left(c_1 z(t) - c_3 x(t) \right) dt$$

$$+ \int_a^b \hat{k} \left(c_1 y(t) - c_2 x(t) \right) dt$$

$$= \hat{i} \left(c_2 \int_a^b z(t)\,dt - c_3 \int_a^b y(t)\,dt \right)$$

$$- \hat{j} \left(c_1 \int_a^b z(t)\,dt - c_3 \int_a^b x(t)\,dt \right)$$

$$+ \hat{k} \left(c_1 \int_a^b y(t)\,dt - c_2 \int_a^b x(t)\,dt \right)$$

$$= \det \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ c_1 & c_2 & c_3 \\ \int_a^b x(t)\,dt & \int_a^b y(t)\,dt & \int_a^b y(t)\,dt \end{pmatrix}$$

$$= \vec{c} \times \int_a^b \vec{r}(t)\,dt.$$

Now, since the integral distributed without fanfare over all the constants, we can "fold" the cross product back up.