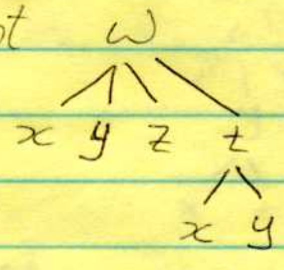
14.9.2) $\omega = x^2 + y - z + \sin t$ and $x + y = t$.

a) $\left(\frac{\partial \omega}{\partial y}\right)_{x,z} = \frac{\partial \omega}{\partial y} + \frac{\partial \omega}{\partial t}\frac{\partial t}{\partial y}$

$= 1 + \cos t$

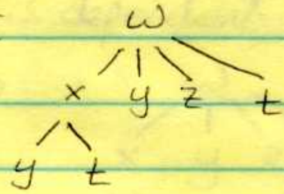
t dependent



b) $\left(\frac{\partial \omega}{\partial y}\right)_{z,t} = \frac{\partial \omega}{\partial x}\frac{\partial x}{\partial y} + \frac{\partial \omega}{\partial y}$

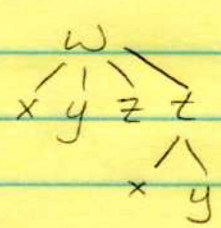
$= 2x(-1) + 1 = 1 - 2x$

x dependent



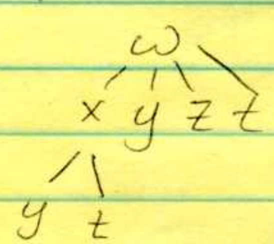
c) $\left(\frac{\partial \omega}{\partial z}\right)_{x,y} = \frac{\partial \omega}{\partial z} \cancel{+ \frac{\partial \omega}{\partial t}\frac{\partial t}{\partial z}} = -1$

t dependent



d) $\left(\frac{\partial \omega}{\partial z}\right)_{y,t} = \frac{\partial \omega}{\partial z} = -1$

x dependent



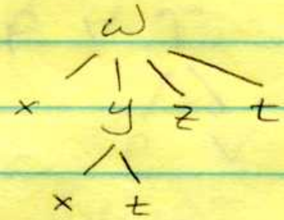
14.9.2) continued

e) $$\left(\frac{\partial w}{\partial t}\right)_{x,z} = \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial w}{\partial t}$$

$$= 1\left(\overset{1}{\cancel{\cos t}}\right) + \cos t = 1 + \cos t$$

y is dependent

$\Rightarrow$

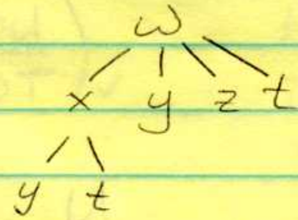


f) $$\left(\frac{\partial w}{\partial t}\right)_{y,z} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial t}$$

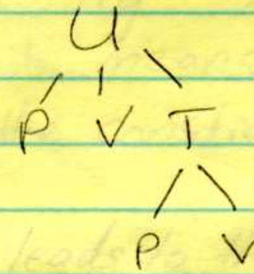
$$= 2x(1) + \cos t$$

x is dependent $= 2x + \cos t$

$\Rightarrow$



14.9.3) $\left(\frac{\partial U}{\partial P}\right)_V$ tells us P, V are independent variables. Therefore, T is dependent.



$$\left(\frac{\partial U}{\partial P}\right)_V = \frac{\partial U}{\partial P} + \frac{\partial U}{\partial T}\frac{\partial T}{\partial P}$$

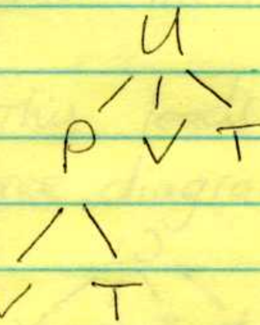
$$= \frac{\partial f}{\partial P} + \frac{\partial f}{\partial T}\left(\frac{V}{nR}\right)$$

Aside: $PV = nRT$

$$T = \frac{PV}{nR}$$

$$\frac{\partial T}{\partial P} = \frac{V}{nR}$$

$\left(\frac{\partial U}{\partial T}\right)_V$ tells us T, V are independent variables. Therefore, P is dependent.



$$\left(\frac{\partial U}{\partial T}\right)_V = \frac{\partial U}{\partial P}\frac{\partial P}{\partial T} + \frac{\partial U}{\partial T}$$

$$= \frac{\partial f}{\partial P}\left(\frac{nR}{V}\right) + \frac{\partial f}{\partial T}.$$

Aside: $PV = nRT$

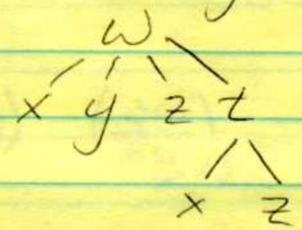
$$P = \frac{nRT}{V}$$

$$\frac{\partial P}{\partial T} = \frac{nR}{V}$$

14.9.8) $w = x^2 - y^2 + 4z + t$ AND $x + 2z + t = 25$.

Let's try x, y, z as the independent variables, which means t is dependent (only on x & z, as the constraint is $x + 2z + t = 25$).

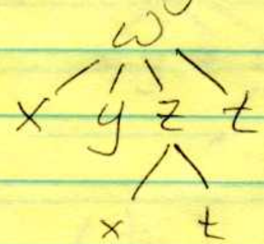
This leads to the tree diagram:



$$\left(\frac{\partial w}{\partial x}\right)_{y,z} = \frac{\partial w}{\partial x} + \frac{\partial w}{\partial t}\frac{\partial t}{\partial x}$$
$$= 2x + (1)(-1)$$
$$= 2x - 1.$$

Now let's try x, y, t as the independent variables, which means z is the dependent variable.

This leads to the tree diagram



$$\left(\frac{\partial w}{\partial x}\right)_{y,t} = \frac{\partial w}{\partial x} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial x}$$
$$= 2x + (4)\left(-\tfrac{1}{2}\right)$$
$$= 2x - 2.$$