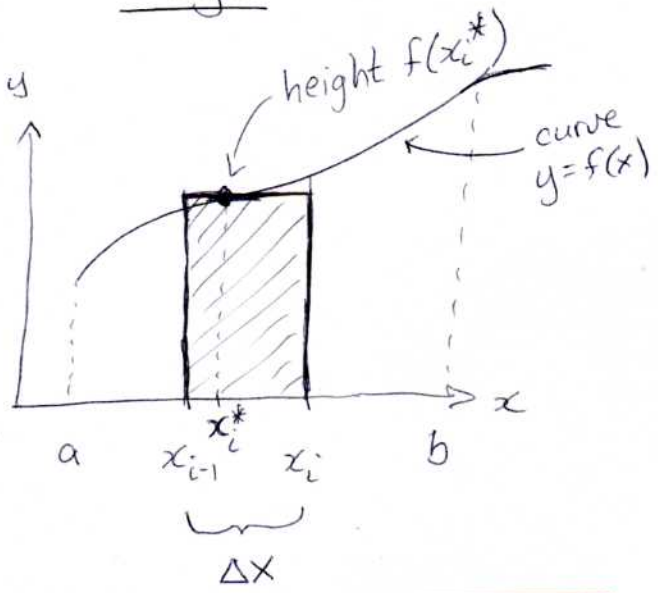
Comparison of Single & Double Integrals

Single

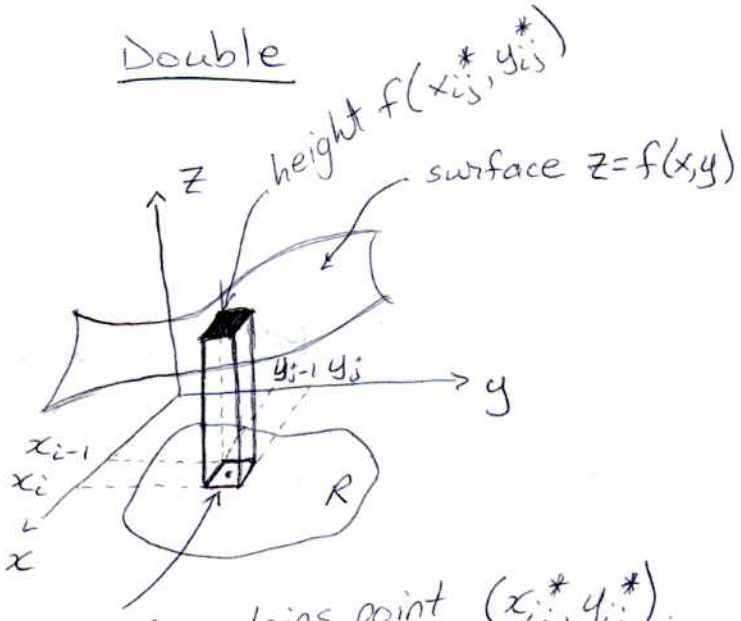


- Δx is width of subinterval
- x_i^* is any point in subinterval
- integral is related to area
- Area of rectangle $\sim$ area under curve

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$ is the average value of $f(x)$ on $[a,b]$

Double



ΔA contains point (x_{ij}^*, y_{ij}^*).

- ΔA is area of subrectangle
- (x_{ij}^*, y_{ij}^*) is any point in subrectangle
- double integral is related to volume
- Volume of rectangular box $\sim$ Volume under curve.

$$\iint_R f(x,y)\,dA = \lim_{n,m\to\infty} \sum_{i=1}^{n}\sum_{j=1}^{m} f(x_{ij}^*, y_{ij}^*)\,\Delta A$$

Note: • $\Delta A = \Delta x\,\Delta y$ so $dA = dx\,dy$

• The region R can be quite complicated.

$f_{ave} = \frac{1}{A(R)}\iint_R f(x,y)\,dA$ is the average value of $f(x,y)$ on region R.
$A(R)$ = Area of R.