This is a sample of the type of questions to expect on the final exam. The final will only cover questions from Chapter 16, but as you can see you are going to have to have a great familiarity with the previous work to solve these problems. So Chapters 12–15 will not appear on the final explicitly, but you will certainly need to know that material!

The final will be 7 or 8 questions long. You should also be prepared to answer questions similar to homework questions, and examples in the text. In other words, this list of questions is by no means complete!

Only the following Useful Relations will be provided on the final:

- Green's Theorem: $\oint_C \mathbf{F} \cdot d\mathbf{r} = \oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dx\,dy$ where $C = \partial R, \mathbf{F} = \langle M, N, 0 \rangle$.
- Stokes' Theorem: $\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\,d\sigma, C : \mathbf{r}(t)$ is the space curve boundary of S.
- Divergence Theorem: $\iint_S \mathbf{F} \cdot \mathbf{n}\,d\sigma = \iiint_E \nabla \cdot \mathbf{F}\,dV$, S is the boundary surface of E, with positive orientation.

Sample Questions

1. (a.) Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y) = \langle 2xy, x^2 \rangle$, and the curve C is given by $\mathbf{r}(t) = t^2\mathbf{i} + t^3\mathbf{j}$, $0 \leq t \leq 1$.

   (b.) Evaluate the line integral $\int_C xy\,dx - x\,dy$ over the curve $y = 1 - x^2$ from the point (1,0) to the point (0,1).

2. Find a function f such that $\mathbf{F} = \nabla f$ and use it to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the curve C.
   $\mathbf{F}(x, y, z) = 4xe^z\mathbf{i} + \cos y\mathbf{j} + 2x^2e^z\mathbf{k}$; $C : \mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + t^4\mathbf{k}$; $0 \leq t \leq 1$.

3. Use Green's Theorem to evaluate the line integral along the given positively oriented curve:
   $\int_C (x^3 - y^3)dx + (x^3 + y^3)dy$, C is the boundary of the region between the curves $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$.

4. (a.) For what value of the constant b is the vector field $\mathbf{F}(x, y, z) = 2x\mathbf{i} + z\mathbf{j} + bz\mathbf{k}$ incompressible?

   (b.) For what value of the constant b is the vector field $\mathbf{F}(x, y, z) = bxy^2\mathbf{i} + x^2y\mathbf{j}$ irrotational?

5. Find the surface area of the part of the paraboloid $x = y^2 + z^2$ that lies inside the cylinder $y^2 + z^2 = 9$.

6. Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\,d\sigma$ for the vector field $\mathbf{F} = -y\mathbf{i} + x\mathbf{j} + 3z\mathbf{k}$, where S is the hemisphere $z = \sqrt{16 - x^2 - y^2}$ with upwards orientation.

7. Show using Stokes' Theorem that the $\iint_S \operatorname{curl} \mathbf{F} \cdot \mathbf{n}\,d\sigma$ where $\mathbf{F}(x, y, z) = \langle y^2z, xz, x^2y^2 \rangle$ and S is the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 1$ oriented upward is $\int_0^{2\pi} (\sin^3 t + \cos^2 t)\,dt$. What would change if the orientation was downwards?

8. Let $\mathbf{F}(x, y, z) = \langle 3xy, y^2, -x^2y^4 \rangle$ and let S be the surface of the tetrahedron with vertices (0,0,0), (1,0,0), (0,1,0), and (0,0,1). Evaluate the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\,d\sigma$.

Answers: 1. 1, $-11/12$ 2. $2e + \sin 1$ 3. 120π 4. $b = -2$, $b = 1$ 5. $\pi(37\sqrt{37} - 1)/6$ 6. 128π 7. change: $\int_{2\pi}^{0}$ 8. $5/24$