Math 2101 Calculus III
Test #1 Practice

This is a sample of the type of questions to expect on the test. The test will be 5 questions long (some questions will have multiple parts). You should also be prepared to answer questions similar to homework questions, and examples in the test. In other words, this list of questions is by no means complete!

You will not be allowed to bring in calculators on the test, but you can bring in one sheet of paper with anything that you care to write on it.

We will go over the solution to some of these problems during the review the day before the test.

1. Determine whether the following two lines are skew, parallel, or intersect. If they intersect, find the point of intersection.

$$L_1 : x = 1 - 2t, y = 1 + 2t, z = -1 + 4t$$
$$L_2 : x = 0, y = 3 + 2s, z = -4 - 4s$$

2. Find the equation of the plane that passes through the point $(3, 4, 1)$ and is parallel to the vector $\langle -1, -1, 1\rangle$.
3. Find the distance from the plane $3x + 4y - z = 14$ to the point $(2, 2, 1)$.
4. Find the projection of $\mathbf{u} = \langle 1, -4, 14\rangle$ in the direction of $\mathbf{v} = \langle 9, -4, 1\rangle$.
5. What is the area of the triangle created by the points $P(1, 2, 3)$, $Q(-1, 0, 2)$ and $R(-1, 2, 2)$?
6. What is the angle between the two planes $x + y + z = 0$ and $x - y - z = 2$?
7. If $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = \mathbf{0}$ and $\mathbf{v} \times \mathbf{w} \neq \mathbf{0}$, then is it true that $\mathbf{u} \perp (\mathbf{v} + \mathbf{w})$? Explain your answer.
8. What is the arc length of the curve $\mathbf{r}(t) = \cos 2t\mathbf{i} + \sin 2t\mathbf{j} + t\mathbf{k}$ from $(1, 0, 0)$ to $(1/\sqrt{2}, 1/\sqrt{2}, \pi/8)$?
9. A particle moves in $\mathbb{R}^3$ so that its velocity and position vectors are always orthogonal. Show that the particle moves on a sphere centered at the origin.
10. Suppose C is the curve given by the vector function $\mathbf{r}(t) = \langle t, t^2, 1 - t^2\rangle$. Find the unit tangent vector, the unit normal vector, and the curvature of C at the point where $t = 1$.
11. The curves $\mathbf{r}_1(t) = \langle t, t^2, t^3\rangle$ and $\mathbf{r}_2(t) = \langle \sin t, \sin 2t, t\rangle$ intersect at the origin. Find the angle of intersection.
12. Find the center of the osculating circle of the parabola $y = x^2$ at the origin.
13. For $\mathbf{r}(t) = t^2\mathbf{i} + t\mathbf{j}$ find the tangential and normal components of the acceleration.
14. Derive $\mathbf{r}(t) = \mathbf{r}_0 + t\,\mathbf{v}_1 + s\,\mathbf{v}_2$, which is the parametric equation for a plane containing two non-parallel vectors $\mathbf{v}_1$ and $\mathbf{v}_2$. You should include a well-labeled sketch in your answer.