Math 2101 Calculus III
Test #3 Practice

The test will be 5 questions long (some questions will have multiple parts). One question will be handed out Wednesday and is a take-home problem (use resources you like, but do not discuss the problem with ANYONE). The in-class portion of the test will be 4 questions.

This is a sample of the type of questions to expect on the test. You should also be prepared to answer questions similar to homework questions, and examples in the test. In other words, this list of questions is by no means complete! You will not be allowed to bring in calculators on the test.

1. (a.) Evaluate the iterated integral $\displaystyle\int_0^1 \int_0^z \int_0^{y^2} x\,dx\,dy\,dz$.

   (b.) Convert to polar and evaluate the integral $\displaystyle\int_0^2 \int_{-\sqrt{4-x^2}}^0 \frac{xy}{\sqrt{x^2+y^2}}\,dy\,dx$.

2. Find the average value of $f(x,y) = xy$ over the region $D = \{(x,y)|x^2+y^2 \le 1, xy \ge 0\}$.

3. Sketch the geometry of the region in the first octant beneath the plane $x + 2y + 3z = 6$. Find the volume of this region using triple integrals.

4. Use spherical coordinates to evaluate $\displaystyle\iiint_E (x^2+y^2+z^2)^2\,dV$, where E is the solid in the first octant bounded by the sphere $x^2+y^2+z^2 = 4$.

5. If E is the solid that lies beneath the paraboloid $z = 1 - x^2 - y^2$ in the first octant, determine $\displaystyle\iiint_E x\,dV$ using cylindrical coordinates.

6. The joint density function for a pair of random variables X and Y is

   $$f(x,y) = \begin{cases} \frac{x}{2}(1+y) & \text{if } 0 \le x \le 1, 0 \le y \le 2 \\ 0 & \text{otherwise} \end{cases}$$

   (a.) Find $P(X \le 1, Y \ge 1)$. (b.) Find $P(X + Y \le 1)$.

   Note the probability is given as the integral of $f(x,y)$ over the region R given by the conditions of the probability.

7. Use a change of variables to calculate $\displaystyle\iint_R e^{9x^2+4y^2}\,dA$, where R is the region in the first quadrant bounded by the ellipse $9x^2+4y^2 = 1$.

8. Using the Jacobian of the cylindrical–cartesian coordinate transformation, derive the formula for triple integration in cylindrical coordinates.

9. Find a triple integral expression for the volume of the solid tetrahedron with vertices (0,0,0),(0,0,1),(0,2,0),(2,2,0). Note: this is a neat question, since you have to work out the equation of a plane!

10. Evaluate $\displaystyle\iint_R \ln\left(\sqrt{x^2+y^2}\right)\,dA$ where $R = \left\{(x,y)|x^2+y^2 \le 1\right\}$.

11. Use cylindrical coordinates to evaluate $\displaystyle\iiint_E x\,dV$, where E is the solid that lies within the cylinder $x^2+y^2 = 1$, above the paraboloid $z = x^2+y^2$, and below the cone $z^2 = 4x^2+4y^2$.

Answers: 1. $1/60$, $-4/3$ 2. $(2\pi)^{-1}$ 3. 6 4. $64\pi/7$ 5. $2/15$ 6. $5/2$, $5/48$ 7. $\pi/24(e-1)$ 9. $\int_0^2 \int_x^2 \int_0^{(2-y)/2} dzdydx$ 10. $-\pi/2$ 11. 0