I am demanding that solutions be written up well. This means solutions should be a self-contained document. They should be written legibly, contain diagrams or tables where appropriate, and should state the problem and explain the solution. Interspersing English sentences which explain what you are doing can help in this regard. With its worked-out examples, the book provides many examples of a good solution. To say it a different way, *solutions with totally correct computations lacking in necessary good explanations will tend to receive a B, not an A, grade.*

You should use *Mathematica* or hand calculations to verify your solutions. If you use *Mathematica* to do integrations, simply state that you have used *Mathematica*. If you have used a significant amount of *Mathematica* in your solution, please print out the *Mathematica* notebook and include it as part of your solution. Remember that you will not be using *Mathematica* on tests, and you may be required to perform integrations using parts, partial fractions, etc. by hand.

Keep in mind when writing up solutions that I am interested in seeing the thought process that you used to solve the problem! If you have any questions about what constitutes a well written solution, come and talk to me. I am more than willing to help you with your assignments so that they serve a useful role in meeting the goals I have set out for us.

Questions

(30) 1. For the differential equation $y' = \dfrac{x^2}{y(1+x^3)}$.

(a) Solve by hand.

(b) Sketch the solution you found in (a) and clearly indicate how any restrictions on the solution arise.

(c) Solve and sketch the solution to the initial value problem $y' = \dfrac{x^2}{y(1+x^3)}, \quad y(0) = 1/2$.

This problem gives you practice with nonlinear differential equations.

(20) 2. The *Gompertz model* $x' = \beta x \ln\left(\dfrac{K}{x}\right), \quad x(0) = x_0$ is used to model tumor growth.

(a) Read up to Equation 4 in the article The effect of correlated noise in a Gompertz tumor growth model by Behera and O'Rourke in the Brazilian Journal of Physics, 2008, vol.38 pp. 272-278 for more background on this application. Give a short explanation of this model.

(b) Solve the Gompertz model initial value problem for $x(t)$ (explicit solution). Verify that the solution satisfies $\lim_{t\to\infty} x(t) = K$.

This problem involves an interesting model.

(30) 3. Consider the initial value problem $\dfrac{dy}{dt} = \dfrac{t\cos t}{1-y^2}, \quad y(0) = -1/2$.

(a.) Solve the initial value problem.

(b.) Sketch the implicit function you found in part (a.). Is this entire implicit function actually a solution to the IVP? Explain.

Hint: Think about the Existence and Uniqueness Theorem; it also might help to complete part (c.) and see the numerical solution to the IVP.

(c.) Use Taylor's method of order 2 to get a numerical solution to the IVP (you can modify the code on the course webpage to do this). Overlay a sketch of the numerical solution with the sketch from part (b.). Discuss your findings, especially with respect to the how long the numerical solution remains a reasonable approximation to the solution.

This problem gives you practice with numerical solutions to ODEs.

(20) 4. Assume that p and q are continuous and that the functions y_1 and y_2 are solutions to the differential equation $y'' + p(t)y' + q(t)y = 0$ on an open interval I. Prove that if y_1 and y_2 have a common point of inflection t_0 in I, then they cannot be a fundamental set of solutions unless both p and q are zero at t_0.

Hint: You know that if $y_1(t_0) = y_2(t_0) = 0$ **or** $y_1'(t_0) = y_2'(t_0) = 0$ then y_1, y_2 do not form a fundamental set of solutions (Wronskian for these two cases is zero). For this problem, work *from the differential equation*, and show that y_1, y_2 are **not** a fundamental set of solutions when

i. $q(t_0) \neq 0$ and $p(t_0) = 0$,

ii. $q(t_0) = 0$ and $p(t_0) \neq 0$,

iii. $q(t_0) \neq 0$ and $p(t_0) \neq 0$.

This problem gives you the chance to work on a more theoretical problem.