Math 2401: DE Assignment 4

(25) 1. Determine the solution to the IVP $y'' + y = \displaystyle\sum_{k=1}^{\infty} k\,\delta(t-k)$, $y(0) = y'(0) = 1$ using Laplace transforms and the table of Laplace transforms. Then use *Mathematica* to plot the solution for $0 < t < 20$.

The function $\sum_{k=-\infty}^{\infty} \delta(t-k)$ is known as the Dirac Comb (we've started at $k = 1$ here rather than $k = -\infty$ and modified it slightly) which often appears in electrical engineering problems. It is a train of equally spaced impulse functions. This type of problem is one that Laplace transforms handle especially well. Imagine what would have to be done to solve this without Laplace transforms!

(25) 2. Consider the system

$$\mathbf{x}' = \mathbf{A}\mathbf{x} = \begin{pmatrix} \frac{5}{4} & \frac{3}{4} \\ \\ \frac{3}{4} & \frac{5}{4} \end{pmatrix} \mathbf{x}.$$

a) Find the general solution to the system and describe the behaviour of the solution as $t \to \infty$.

b) Check that the two solutions you find are linearly independent.

c) Use *Mathematica* to create a direction field for the system and plot a few trajectories of the system.

This problem gives you practice with the basic method of solving a system of differential equations by solving the associated eigensystem problem, and the new method of calculating a Wronskian.

(25) 3. Find the general <u>real-valued solution</u> to

$$t\mathbf{x}' = \mathbf{A}\mathbf{x} = \begin{pmatrix} -1 & -1 \\ 4 & -1 \end{pmatrix} \mathbf{x}, \qquad t > 0.$$

Hint: The system $t\mathbf{x}' = \mathbf{A}\mathbf{x}$ is analogous to the Euler equation. Your assumed solution should be suitably modified. You do not have to verify that your solutions form a fundamental set of solutions.

Solving this problem shows you how the concept of an Euler equation extends to a system of differential equations, as well as how to construct real valued solutions from complex solutions.

(25) 4. By computing eigenvalues and eigenvectors, and using undetermined coefficients, solve the IVP

$$\mathbf{x}' = \mathbf{A}\mathbf{x} + \mathbf{g}(t) = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} -4 \\ 1 \end{pmatrix} t.$$

Solving this problem gives you a chance to explore undetermined coefficients applied to a nonhomogeneous system of differential equations.