Section 2.2

Example (2.2.1) Solve the differential equation $y' = x^2/y$.

$$\begin{aligned} \frac{dy}{dx} &= \frac{x^2}{y} \\ y\,dy &= x^2\,dx \\ \int y\,dy &= \int x^2\,dx \\ \frac{y^2}{2} &= \frac{x^3}{3} + c \end{aligned}$$

This is an implicit solution for y.

Example (2.2.4) Solve the differential equation $y' = (3x^2 - 1)/(3 + 2y)$.

$$\begin{aligned} \frac{dy}{dx} &= \frac{3x^2 - 1}{3 + 2y} \\ (3 + 2y)\,dy &= (3x^2 - 1)\,dx \\ \int (3 + 2y)\,dy &= \int (3x^2 - 1)\,dx \\ 3y + y^2 &= x^3 - x + c \end{aligned}$$

This is an implicit solution for y.

Example (2.2.24) Solve the initial value problem $y' = (2 - e^x)/(3 + 2y), y(0) = 0$ and determine where the solution attains its maximum value.

$$\begin{aligned} \frac{dy}{dx} &= \frac{2 - e^x}{3 + 2y} \\ (3 + 2y)\,dy &= (2 - e^x)\,dx \\ \int (3 + 2y)\,dy &= \int (2 - e^x)\,dx \\ 3y + y^2 &= 2x - e^x + c \end{aligned}$$

We use the initial condition $y(0) = 0$ to determine the constant c. When $x = 0$, $y = 0$:

$$\begin{aligned} 0 &= -1 + c \\ c &= 1 \end{aligned}$$

The implicit solution to the initial value problem is $3y + y^2 = 2x - e^x + 1$.

The maximum occurs when $y' = 0$. Implicitly differentiate:

$$\begin{aligned} \frac{d}{dx}[3y + y^2 &= 2x - e^x + c] \\ 3\frac{dy}{dx} + 2y\frac{dy}{dx} &= 2 - e^x \end{aligned}$$

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\begin{aligned}
0 &= 2 - e^x \\
e^x &= 2 \\
x &= \ln 2
\end{aligned}
$$

This is where the maximum occurs.