Section 3.2

Example (3.2.1) Find the Wronskian of e^{2t} and $e^{-3t/2}$.

$$\begin{aligned} W(y_1, y_2)(t) &= \begin{vmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{vmatrix} \\ W(e^{2t}, e^{-3t/2})(t) &= \begin{vmatrix} e^{2t} & e^{-3t/2} \\ 2e^{2t} & -\frac{3}{2}e^{-3t/2} \end{vmatrix} \\ &= -\frac{3}{2}e^{-3t/2}e^{2t} - e^{-3t/2}(2e^{2t}) \\ &= -\frac{7}{2}e^{t/2} \end{aligned}$$

Example (3.2.6) Find the Wronskian of $\cos^2 t$ and $1 + \cos 2t$.

$$\begin{aligned} W(y_1, y_2)(t) &= \begin{vmatrix} y_1(t) & y_2(t) \\ y_1'(t) & y_2'(t) \end{vmatrix} \\ W(\cos^2 t, 1 + \cos 2t)(t) &= \begin{vmatrix} \cos^2 t & 1 + \cos 2t \\ -2\cos t \sin t & -2\sin 2t \end{vmatrix} \\ &= \cos^2 t(-2\sin 2t) - (1 + \cos 2t)(-2\cos t \sin t) \\ &= -2\cos^2 t(2\cos t \sin t) + 2\cos t \sin t + 2\cos t \sin t(\cos^2 t - \sin^2 t) \\ &= -4\cos^3 t \sin t + 2\cos t \sin t + 2\cos^3 t \sin t - 2\cos t \sin^3 t \\ &= -2\cos t \sin t(\cos^2 t + \sin^2 t) + 2\cos t \sin t \\ &= -2\cos t \sin t + 2\cos t \sin t = 0 \end{aligned}$$

Example (3.2.9) Determine the longest interval in which the initial value problem $t(t-4)y'' + 3ty' + 4y = 2, y(3) = 0, y'(3) = -1$ is certain to have a unique twice differentiable solution. Do not attempt to find the solution.

Write the differential equation in standard form: $y'' + \dfrac{3}{t-4}y' + \dfrac{4}{t(t-4)}y = \dfrac{2}{t(t-4)}$.

Identify the important quantities: $p(t) = \dfrac{3}{t-4}$; $q(t) = \dfrac{4}{t(t-4)}$; $g(t) = \dfrac{2}{t(t-4)}$.

The points of discontinuity are $t = 0$ and $t = 4$. The initial point is at $t_0 = 3$, so the the longest interval where p, q, g are continuous is $t \in (0, 4)$. Theorem 3.2.1 tells us a solution will exist on this interval.