Section 6.3 Step Functions

Example (6.3.4) See *Mathematica* notebook for sketch.

Example (6.3.7) Find the Laplace transform of $f(t) = \begin{cases} 0, & t < 2 \\ (t-2)^2, & t \geq 2 \end{cases}$.

We want to write $f(t)$ in terms of $u_c(t)$ so we can use our table of Laplace transforms.

$$f(t) = u_2(t)(t-2)^2.$$

$$\begin{aligned} \mathcal{L}[f(t)] &= \int_0^\infty e^{-st} f(t)\, dt \\ &= \int_0^\infty e^{-st} u_2(t)(t-2)^2\, dt \\ &= \int_2^\infty e^{-st}(t-2)^2\, dt \end{aligned}$$

use *Mathematica* to do the integral, or use parts

$$= \frac{2e^{-2s}}{s^3}, \quad s > 0$$

Example (6.3.18) Find the inverse Laplace transform of $F(s) = \dfrac{e^{-s} + e^{-2s} - e^{-3s} - e^{-4s}}{s}$.

Here we just need to use Table 6.2.1 #12 repeatedly.

$$\begin{aligned} \mathcal{L}[f(t)] &= \mathcal{L}^{-1}[\frac{e^{-s}}{s}] + \mathcal{L}^{-1}[\frac{e^{-2s}}{s}] - \mathcal{L}^{-1}[\frac{e^{-3s}}{s}] - \mathcal{L}^{-1}[\frac{e^{-4s}}{s}] \\ &= u_1(t) + u_2(t) - u_3(t) - u_4(t) \end{aligned}$$