Theorem 1 Existence and Uniqueness Theorem (Theorem 3.2.1) *Consider the initial value problem*

$$y'' + p(t)y' + q(t)y = g(t), \quad y(t_0) = y_0, \quad y'(t_0) = y'_0$$

where p, q and g are continuous on an open interval I. Then there exists exactly one solution $y = \phi(t)$ of this problem, and the solution exists throughout the interval I.

Theorem 2 Principle of Superposition (Theorem 3.2.2) *If y_1 and y_2 are two solutions of the differential equation*

$$L[y] = y'' + p(t)y' + q(t) = 0,$$

then the linear combination $y(t) = c_1y_1 + c_2y_2$ is also a solution for any values of the constants c_1 and c_2.

Theorem 3 (Theorem 3.2.3) *Suppose that y_1 and y_2 are two solutions of*

$$L[y] = y'' + p(t)y' + q(t)y = 0,$$

and that the Wronskian

$$W = y_1y'_2 - y'_1y_2,$$

is not zero at the point t_0 where the initial conditions $y(t_0) = y_0$, $y'(t_0) = y'_0$ are assigned. Then there is a choice of constant c_1 and c_2 for which $y = c_1y_1(t) + c_2y_2(t)$ satisfies the associated IVP.

Theorem 4 (Theorem 3.2.4) *If y_1 and y_2 are two solutions of the DE:*

$$L[y] = y'' + p(t)y' + q(t)y = 0,$$

and if there is a point t_0 where the Wronskian of y_1 and y_2 is nonzero, then the family of solutions

$$y(t) = c_1y_1(t) + c_2y_2(t)$$

with arbitrary coefficients c_1 and c_2 includes every solution of the DE.

Theorem 5 (Theorem 3.2.5) *Consider the DE*

$$L[y] = y'' + p(t)y' + q(t)y = 0,$$

whose coefficients p, q are continuous on some open interval I. Choose some point t_0 in I. Let y_1 be the solution of the DE that also satisfies the initial conditions:

$$y(t_0) = 1, \qquad y'(t_0) = 0,$$

and let y_2 be the solution of the DE that also satisfies the initial conditions:

$$y(t_0) = 0, \qquad y'(t_0) = 1.$$

Then y_1 and y_2 form a fundamental set of solutions for the DE.

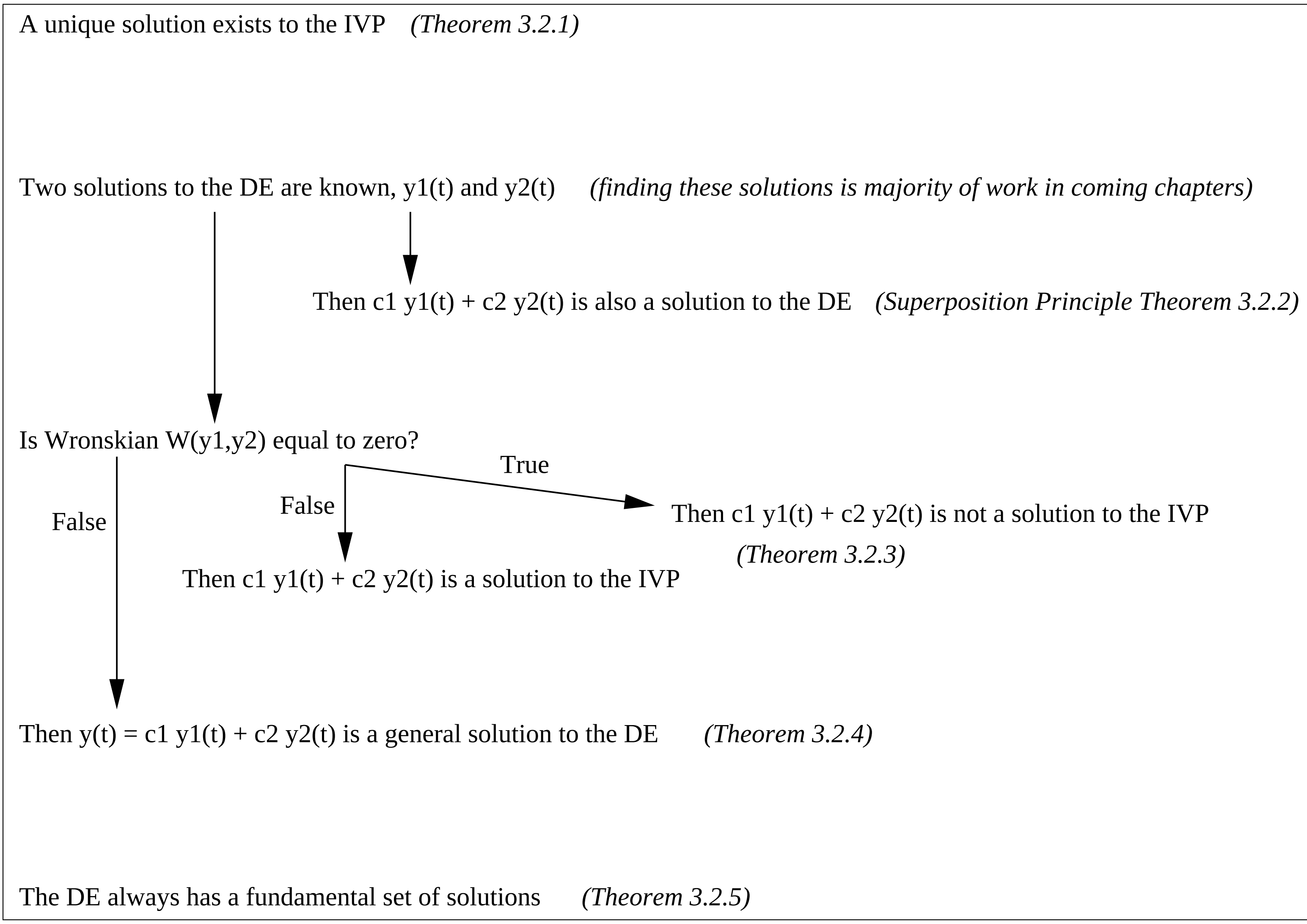
Here is a concept map of how the theorems relate. In what follows we have

DE: $\quad y''(t) + p(t)y'(t) + q(t)y(t) = 0,$

IVP: $\quad y''(t) + p(t)y'(t) + q(t)y(t) = 0, \qquad y(t_0) = y_0, \quad y'(t_0) = y_0'.$

where p, q and g are continuous in an interval I.

Note that Theorem 3.2.1 also applies to nonhomogeneous linear differential equations.



Theorem 6 Abel's Theorem (Theorem 3.2.6) *If y_1 and y_2 are solutions of the differential equation*

$$L[y](t) = y'' + p(t)y' + q(t)y = 0,$$

where p and q are continuous on an open interval I, then the Wronskian $W(y_1, y_2)(t)$ is given by:

$$W(y_1, y_2)(t) = c \exp\left[-\int p(t)dt\right],$$

where c is a certain constant that depends on y_1, y_2, but not on t. Further, $W(y_1, y_2)(t)$ is either zero for all t in I (if $c = 0$) or else is never zero in I (if $c \neq 0$).