You should be *expanding* this study guide as you see fit with details and worked examples. With this extra layer of detail you will then have excellent study notes for exams, and later reference.

Some topics will be emphasized more than others.

Practice is suggested from Boyce and DiPrima *Elementary Differential Equations*, 10th Edition.

1.1-1.4 Introduction: Terminology and Classification of Differential Equations (WEEK OF AUG 24)

- differential equation (DE) and initial value problem (IVP)
- general solution
- equilibrium solution
- ordinary and partial differential equations
- order, linear and nonlinear
- Practice: 1.1: 15-20, 1.2.8, 1.2.15, 1.3: 1-6, 11

2.1 Linear Equations; Method of Integrating Factors (WEEK OF AUG 31)

- general form of linear first order DE $\frac{dy}{dt} + p(t)y = g(t)$
- integrating factor technique
- the general form of the solution to DE
- the solution to IVP
- Practice: 13, 14, 15, 28, 30

2.2 Separable Equations (WEEK OF AUG 31)

- the general form of a first order separable equation (can be nonlinear) $\frac{dy}{dt} = f(y)g(t)$
- integral curves
- solutions to IVP only portion of an implicit function
- Practice: 1-8, 16, 22

2.3 Modeling with First Order Equations (WEEK OF AUG 31)

- the steps in constructing a mathematical model
- fluid flow in a tank
- equilibrium solution
- radiocarbon dating
- Practice: 1, 4, 6, 17

2.4 Differences Between Linear and Nonlinear Equations (WEEK OF AUG 31)

- general and explicit solutions exist for linear first order DE
- general solution may not exist for nonlinear equations
- implicit solutions may be better for nonlinear equations
- graphical solutions can be very helpful!
- Practice:1, 2, 22, 23

2.5 Autonomous Equations and Population Dynamics (WEEK OF SEP 7)

- definition of autonomous DE (can be nonlinear) $\frac{dy}{dt} = f(y)$
- examples are exponential and logistic growth
- equilibrium solutions and critical points
- how to obtain a picture of the solution without solving DE
- intrinsic growth rate, carrying capacity
- asymptotically stable, unstable and semistable solutions
- Practice: 2, 11, 12, 15

2.6 Exact Equations and Integrating Factors (WEEK OF SEP 7)

- the general form of an exact DE (can be nonlinear) $M(x,y) + N(x,y)\frac{dy}{dx} = 0$ where $M_y = N_x$
- using the integrating factor technique to make a DE exact will not be on test
- Practice: 11, 12, 13, 18

2.7 Numerical Approximations: Euler's Method (WEEK OF SEP 14)

- we will use Taylor's Method of Order 2 rather than Euler's Method
- derivation of the method
- you will not be asked to implement the method on the Test
- Practice: 15

2.8 The Existence and Uniqueness Theorem (WEEK OF SEP 14)

- how to derive the integral equation which corresponds to IVP
- method of successive approximations (Picard's Iteration Method)
- Practice: 1, 3

3.1 Homogeneous Equations with Constant Coefficients (WEEK OF SEP 14)

- general form $ay'' + by' + c = 0$
- solution looks like $y = e^{rt}$ for constant coefficient equations only
- characteristic equation
- fundamental set of solutions (summary of possibilities from Section 3.1, 3.3, 3.4)
  - real distinct roots $r_1 \neq r_2$, fundamental set of solutions is $y_1(t) = e^{r_1 t}, y_2(t) = e^{r_2 t}$
  - real repeated roots r_1 multiplicity 2, fundamental set of solutions is $y_1(t) = e^{r_1 t}, y_2(t) = te^{r_1 t}$
  - complex conjugate roots $r = \lambda \pm \mu i$, fundamental set of solutions is $y_1 = e^{rt}\cos\mu t$, $y_2 = e^{rt}\sin\mu t$
- general solution $y(t) = c_1 y_1(t) + c_2 y_2(t)$
- Practice: 3, 21, 22, 25

3.2 Solutions of Linear Homogeneous Equations; The Wronskian (WEEK OF SEP 21)

- Principle of Superposition
- Wronskian
- general solution
- fundamental set of solutions
- Abel's Theorem
- Practice: 6, 7, 10, 28

3.3 Complex Roots of Characteristic Equation (week of sep 21)

- Euler's formula
- basic concepts of the complex number $a + bi$
- complex solutions
- real solutions from complex solutions
- oscillatory behaviour
- Practice: 1, 5, 7, 9, 24

3.4 Repeated Roots; Reduction of Order (week of sep 21)

- characteristic equation has roots of multiplicity two
- the method of reduction of order
- behaviour of the solution
- Practice: 1, 3, 15, 18

3.5 Nonhomogeneous Equations: Method of Undetermined Coefficients (week of sep 28)

- when you may use the method of undetermined coefficients
- how to use the method
- know when the assumed solution should be multiplied by t
- Practice: 1, 5, 21, 29

3.6 Variation of Parameters (week of sep 28)

- in general more powerful than undetermined coefficients
- work from scratch rather than memorize final formulas
- Practice: 1, 5, 13, 21

3.7 Application: Mechanical and Electrical Vibrations (week of sep 28)

- the meaning of the parameters in $mu''(t) + \gamma u'(t) + ku(t) = F(t)$
- undamped free vibrations $mu''(t) + ku(t) = 0$
  - natural frequency $\omega_0 = \sqrt{k/m}$
  - amplitude R
  - phase δ
- damped free vibrations
  - critical damping (repeated roots of characteristic equation)
  - overdamping (real distinct roots)
  - underdamping (complex roots)
- we will probably not look at electrical circuits, but it you are a physics major read that section
- Practice: 15, 16, 19

3.8 Application: Forced Vibrations (time permitting) (week of oct 5)

- $mu''(t) + \gamma u'(t) + ku(t) = F(t)$
- resonance
- beats
- transient solution
- steady state solution
- phase plane
- Practice: 5, 6, 7, 8

4.1 General Theory of n^{th} Order Linear Equations (week of oct 12)

- general form
- homogeneous, nonhomogeneous, linear, nonlinear
- characteristic equation
- general solution
- linear dependence and linear independence
- Practice: 1, 7, 11, 20

4.2 Homogeneous Equations with Constant Coefficients (week of oct 12)

- characteristic equation
- real and unequal roots
- complex roots
- repeated roots
- Practice: 11, 29, 37, 39

4.3 The Method of Undetermined Coefficients (week of oct 12)

- Practice: 1, 2, 9, 13

4.4 The Method of Variation of Parameters (week of oct 12)

- Practice: 1, 2, 9, 14

7.1 Introduction & 7.2 Review of Matrices (week of oct 12)

- matrix addition, subtraction, multiplication
- matrix times a vector
- identity matrix
- there is more in 7.2 than we need, refer to this section as needed
- Practice: 7.1: 1, 5, 8, 9

7.3 Systems of Algebraic Equations; Linear Independence; Eigenvalues and Eigenvectors (WEEK OF OCT 19)

- homogeneous and nonhomogeneous equations
- systems in linear algebra
- eigenvalues and eigenvectors of an equation
- linear independence
- Practice: 1, 5, 7, 16, 22

7.4 Basic Theory of Systems of First Order Linear Equations (WEEK OF OCT 19)

- relation to second order equations
- Principle of Superposition
- Wronskian
- general solution
- fundamental set of solutions
- Abel's Theorem
- Practice: 2, 6

7.5 Homogeneous Linear Systems with Constant Coefficients (WEEK OF OCT 26)

- phase plane
- phase portrait
- proper nodes, saddle points
- Practice: 1, 7, 13, 15

7.6 Complex Eigenvalues (WEEK OF OCT 26)

- complex solutions
- real valued solutions from the complex
- spiral points
- Practice: 1, 7, 9, 13

7.8 Repeated Eigenvalues (WEEK OF OCT 26)

- second solution from first
- improper nodes
- Practice: 1, 7, 11

7.9 Nonhomogeneous Linear Systems (WEEK OF OCT 26)

- undetermined coefficients
- diagonalization and variation will not be used for linear systems
- Practice: 1, 3, 7

9.1 The Phase Plane: Linear Systems (WEEK OF NOV 2)

- the importance of the point $(x_1, x_2) = (0, 0)$
- nodal sink or source (real unequal eigenvalues of same sign)
- saddle point (real unequal eigenvalues of opposite sign)
- proper node (equal eigenvalues)
- one independent eigenvalue (improper node)
- complex eigenvalues (spiral source or sink)
- you should be able to relate the above to graphs of solutions
- Practice: 1, 3, 13, 20

9.2 Autonomous Systems and Stability (WEEK OF NOV 9)

- autonomous equations
- critical points
- stand and unstable
- basin of attraction
- Practice: 1, 5, 12

9.3 Locally Linear Systems (WEEK OF NOV 9)

- effect of small perturbation
- isolated critical point
- locally linear
- Jacobian matrix
- Practice: 1, 2, 3, 5

9.4 Competing Species (WEEK OF NOV 9)

- nullclines
- Practice: 1, 2, 3, 9

9.5 Predator-Prey Equations (time permitting) (WEEK OF NOV 9)

- derivation of model equations
- Practice: 1, 5, 7

6.1 Definition of Laplace Transform (WEEK OF NOV 9)

- exponential order
- linear operator
- Practice: 1, 5, 7, 15

6.2 Solutions of Initial Value Problems (week of nov 9)

- improper integrals & partial fractions
- exponential order
- properties of Laplace transform: linearity, uniqueness
- inverse Laplace transform
- Practice: 1, 5, 11, 28

6.3 Step Functions (week of nov 16)

- unit step function (heaviside step function)
- piecewise functions in terms of Heaviside step function
- Laplace transform of Heaviside step function
- Practice: 1, 7, 19

6.4 Differential Equations with Discontinuous Forcing Function (week of nov 16)

- Practice: 1, 2, 5, 14

6.5 Impulse Functions (week of nov 16)

- Dirac delta function
- Practice: 1, 2, 5, 14

6.6 The Convolution Integral (week of nov 23)

- Practice: 1, 2, 3, 12, 13

5.1 Review of Power Series (week of nov 23)

- radius and interval of convergence
- analytic functions
- shift index of summation
- manipulation series
- Practice: 8, 11, 14, 18

5.2 & 5.3 Series Solution near an Ordinary Point (week of nov 30)

- definition of an ordinary point and singular point
- the recurrence relation
- the assumed solution
- Practice: 5.2: 1, 3, 5, 19; 5.3: 1, 2, 11

5.4 Euler Equations; Regular Singular Points (WEEK OF NOV 30)

- definition of regular and irregular singular points
- singularity at infinity
- assumed solution to an Euler equation
- real distinct roots, equal roots, complex roots
- Practice: 1, 3, 5, 13, 23

5.5 & 5.6 Series Solution Near Regular Singular Point (WEEK OF DEC 7)

- indicial equation (exponents at the singularity)
- the recurrence relation
- equal roots of the indicial equation use reduction of order
- for roots differing by an integer things get complicated
- Practice: 5.5: 1, 3; 5.6: 1, 13

5.8 Bessel's Equation (time permitting) (WEEK OF DEC 7)

- Practice: 1, 2