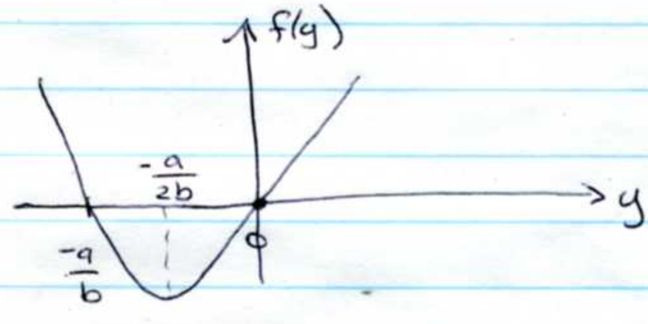
2.5.1 $\frac{dy}{dt} = ay + by^2, \quad a>0, b>0, \ y_0 \geq 0.$

how boring. I'm doing all y_0.

First, sketch $f(y) = ay + by^2 = y(a+by)$:

zero at $y=0$ $y = -a/b$.
parabola opening up.
hump is at $\frac{-a}{2b}$ (since it's a parabola).

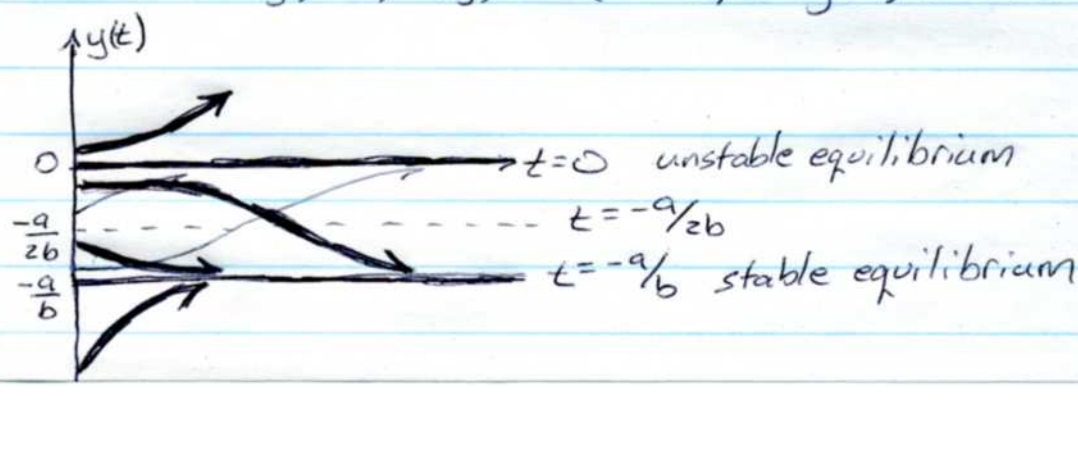


Use this to sketch $y(t)$:

since $f(y)=0$ for $y=0, y=-a/b$ the equilibrium solutions are $y(t)=0$ and $y(t)=-a/b$.

Interval	Facts	what the facts tell us
$-a/b < y < 0$	$f(y) < 0$	$y(t)$ decreasing
$-\infty < y < -\frac{a}{b}$ and $y > 0$	$f(y) > 0$	$y(t)$ increasing
$-\infty < y < -\frac{a}{b}$	$f(y)>0, f'(y)<0$ (opposite)	$y(t)$ concave down
$-\frac{a}{b} < y < -\frac{a}{2b}$	$f(y)<0, f'(y)<0$ (same)	$y(t)$ concave up
$-\frac{a}{2b} < y < 0$	$f(y)<0, f'(y)>0$ (opposite)	$y(t)$ concave down
$0 < y < \infty$	$f(y)>0, f'(y)>0$ (same)	$y(t)$ concave up.

sketch:



2.5.3 $\frac{dy}{dt} = y(y-1)(y-2)$

First, sketch $f(y) = y(y-1)(y-2)$

zero at $y=0, y=1, y=2$

cubic, so it will have two humps.

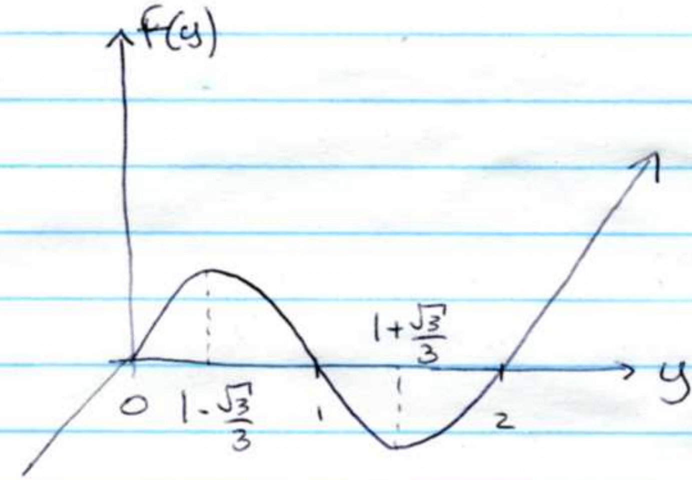
$f(y) = y^3 - 3y^2 + 2y$

$\frac{df}{dy} = 3y^2 - 6y + 2 = 0$ for location of max/min.

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{36-24}}{6} = 1 \pm \frac{\sqrt{3}}{3}$$

As $y \to \infty$, $f(y) \to \infty(\infty)(\infty) = \infty$.



Use this to sketch $y(t)$.

since $f(y) = 0$ for $y = 0, 1, 2$, the equilibrium solutions are $y(t) = 0$

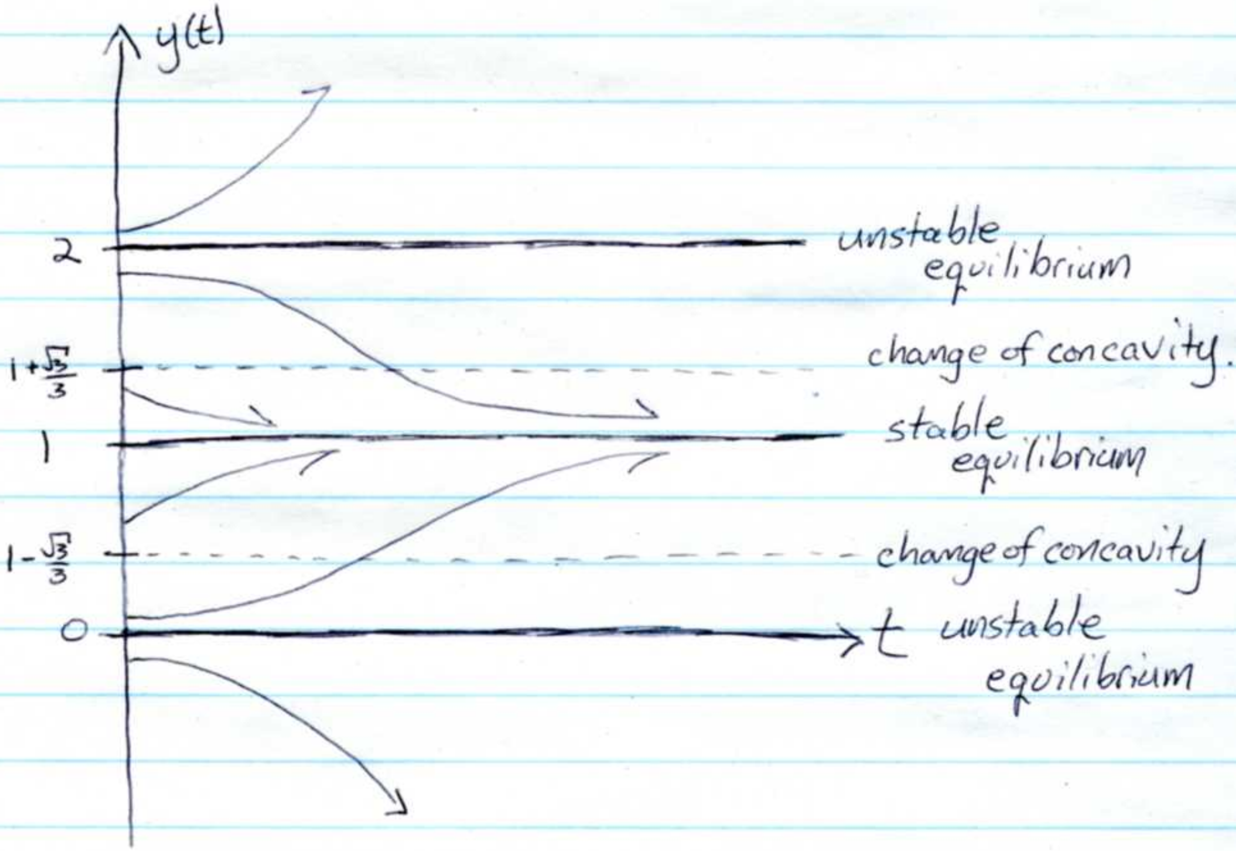
$y(t) = 1$

$y(t) = 2$

2.5.3 continued

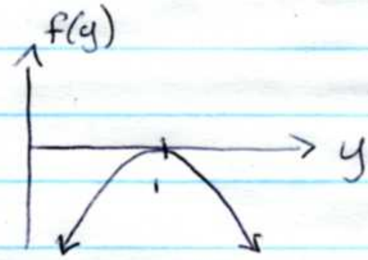
Interval	Facts	What the facts tell us
$-\infty < y < 0$	$f(y) < 0$	$y(t)$ decreasing
$0 < y < 1$	$f(y) > 0$	$y(t)$ increasing
$1 < y < 2$	$f(y) < 0$	$y(t)$ decreasing
$2 < y < \infty$	$f(y) > 0$	$y(t)$ increasing

$-\infty < y < 0$	$f(y) < 0, f'(y) > 0$ (opposite)	$y(t)$ concave down
$0 < y < 1 - \frac{\sqrt{3}}{3}$	$f(y) > 0, f'(y) > 0$ (same)	$y(t)$ concave up
$1 - \frac{\sqrt{3}}{3} < y < 1$	$f(y) > 0, f'(y) < 0$ (opposite)	$y(t)$ concave down
$1 < y < 1 + \frac{\sqrt{3}}{3}$	$f(y) < 0, f'(y) < 0$ (same)	$y(t)$ concave up
$1 + \frac{\sqrt{3}}{3} < y < 2$	$f(y) < 0, f'(y) > 0$ (opposite)	$y(t)$ concave down
$2 < y < \infty$	$f(y) > 0, f'(y) > 0$ (same)	$y(t)$ concave up.



2.5.8) $\frac{dy}{dt} = -k(y-1)^2$, $k > 0$, $-\infty < y_0 < \infty$

First, sketch $f(y) = -k(y-1)^2$



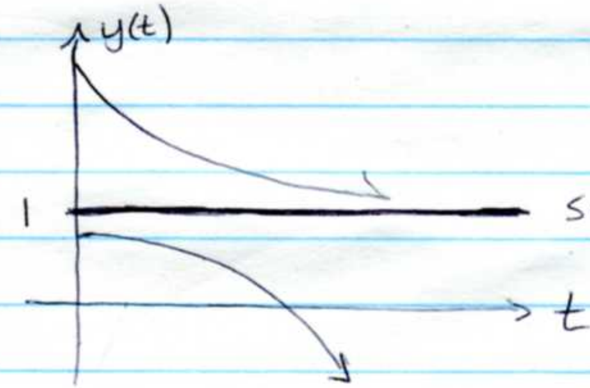
Zero at $y=1$ of multiplicity 2
parabola opening down.

Use this to sketch $y(t)$:

Since $f(y)=0$ at $y=1$, the only equilibrium solution is $y(t)=1$.

Interval	Facts	What the facts tell us
$-\infty < y < \infty$	$f(y) < 0$	$y(t)$ is decreasing
$-\infty < y < 1$	$f(y) < 0$, $f'(y) > 0$ (opposite)	$y(t)$ concave down
$1 < y < \infty$	$f(y) < 0$, $f'(y) < 0$ (same)	$y(t)$ concave up

sketch:



semistable equilibrium.